ENOC 2017, June 25-30, 2017, Budapest, Hungary

Intermittent Oscillations of Elastic structure in Fluctuating Axial Fluid Flow

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Summary. Limit cycle oscillations (LCOs), which result from a Hopf bifurcation, have been a subject of intense study in the field of aeroelasticity. However, it has been recently observed that the presence of fluctuation in the input wind can change the scenario and intermittency can precede the onset of LCOs. In this work, the appearance of intermittency in the oscillation of a flexible plate in fluctuating incoming flow is reported. It has been seen that intermittency occurs only when fluctuations have large temporal correlations. To explain the phenomena, the role of plate flexibility and wake vortical accumulation in the manifestation of intermittency is investigated.

Introduction

Flexible structures such as thin plates, blades, airfoils immersed in axial fluid flow are known to lose dynamic stability through a Hopf bifurcation leading to limit cycle oscillations when the flow exceeds a critical velocity. Similar loss of dynamics stability, also called as flutter, has been observed in aeroelastic systems like aircraft wings under bending and torsion (pitch-plunge airfoils). The flutter instability leads to sustained oscillations and is undesirable as it leads to loss of structural integrity [1]. Recent studies have however focussed on using this instability for energy harvesting [2, 3, 4]. There is therefore a need to ascertain the Hopf bifurcation point accurately. Studies available in the literature typically consider the flow to be steady and uniform. However, recent studies have shown that random fluctuations in the flow have the potential to alter the dynamical stabulity characteristics leading to intermittent oscillations at mean flow speeds significantly smaller than the flutter velocity. Such responses involve switching between high amplitude periodic oscillations and low amplitude periodic/aperiodic oscillations, over time. Similar investigations have not yet been conducted systematically for plate like structures, which are much more agile and flexible compared to bending-twisting wing models.

In this study, the vibratory response of a thin flexible cantilever plate in fluctuating axial fluid flow is investigated. It is seen that intermittency precedes LCOs when the flow fluctuations have significant temporal correlation. To the best of the authors' knowledge, this has not been reported in literature for continuous (*i.e.* the flexibility along flow direction being non-trivial) aeroelastic systems. Since the behaviour of the system is governed by interaction between the plate and the fluid vortices- the bound vortices moving with the plate and the vortices shed into the wake- the role of this interaction in the appearance of intermittency is explored.

Modelling Fluid Structure Interaction

A flat cantilevered, flexible thin plate fixed at the leading edge and free at the trailing edge placed in an axial fluid flow is considered and modelled following Tang and Paidoussis [4]. The assumptions of finite length and infinite width of the plate are made so that the investigations can be restricted to a two-dimensional analysis. A nonlinear structural model following Euler-Bernoulli beam theory has been used accounting for large oscillations in the plate. The fluid loads on the plate are calculated through an unsteady lumped vortex model, considering flow to be inviscid and incompressible with discrete lumped vortices. The bound vortices on the panels and instantaneously formed wake vortices together contribute to the lift and drag forces. The non-dimensional equations of motion [4] are given by

$$\begin{aligned}
&\ddot{w} + \left(1+\alpha\frac{\partial}{\partial\tau}\right)[w''''(1+w'^2) + 4w'w''w''' + w'^3] + w'\int_0^s(\dot{w}'^2 + w'\ddot{w}')ds \\
&-w''\int_s^1\left[\int_0^s(\dot{w}'^2 + w'\ddot{w}')ds\right]ds = \mu U_R^2\left(f_L - w'f_D + w''\int_s^1 f_D ds\right), \qquad (1) \\
&v = -\tfrac{1}{2}\int_0^s w'^2 ds.
\end{aligned}$$

Here, w is the non-dimensional transverse displacement, v is the non-dimensional longitudinal displacement, s is non-dimensional coordinate along the plate centerline, τ is the non-dimensional time, μ is the mass ratio, α is the non-dimensional material damping coefficient, U_R is non-dimensional flow velocity and f_L and f_D are the lift and drag forces respectively [4]. Eqn. (1) was numerically solved using Galerkin expansion of w and Houbolt's method in MATLAB for 200 panels, 2 modes, using a time step size of 0.001 and a non-dimensional truncated wake street length of 40. The other parameters considered were $\mu = 0.2$ and $\alpha = 0.004$.

Modelling random flow using Karhunen-Loeve Expansion

The flow fluctuations is simulated through Karhunen-Loeve expansion (KLE), which involves a bi-orthogonal decomposition of the temporal correlation function of the axial flow velocity. The flow velocity U_R is expanded using KLE as,

$$U_R(\tau, \theta) = \sum_{i \geq 1} \sqrt{\lambda_i} u_i(\tau) \eta_i(\theta), \tag{2}$$

where, τ is the non-dimensional time, λ_i and $u_i(\tau)$ are the eigenvalues and eigenvectors of the correlation function of the process $U_R(\tau, \theta)$ and $\eta_i(\theta)$ are zero mean, unit variance and mutually uncorrelated random variables. In this study, the fluctuating fluid flow is assumed to be a Gaussian process with an exponential auto-correlation function $R_{UU}(\tau_{lag}) = \sigma^2 \exp(-c\tau_{lag}^2)$, where, σ^2 is the variance of the process, c is a scalar value dependent on the correlation time and τ_{lag} is the time lag for which correlation is calculated. The values for σ, and c have been taken to be 0.5 and 0.1 respectively. The mean non-dimensional flow velocity was taken to be 8. The eigenvectors of the correlation matrix thus obtained and Gaussian random variables $\eta_i(\theta)$ generated through pseudo-random number generator together provide the bi-orthogonal bases using which the KLE of the inflow fluctuations is constructed.

Results and Discussion

Figure 1a shows the bifurcation diagram for the system when the flow is free of fluctuations, *i.e.* the inflow is steady and uniform. A Hopf bifurcation occurs at $U_R = 9$ accompanied by onset of LCOs. A time history of the LCOs at $U_R = 10$ is shown in Fig.1b, which indicates oscillations with sustained amplitude. On the contrary, in the presence of flow fluctuations intermittent responses occur before the onset of LCOs, as shown in Fig.2. Interestingly, such oscillations were observed only under the action of flow fluctuations with large correlations. For rapidly varying fluctuations with low temporal correlation but comparable fluctuation intensity, intermittency did not occur. It was also observed that this dichotomy can be explained to be a result of the accumulation of vortical effects due to the effect of plate flexibility and wake. Fluctuations with low correlation seem to require having much higher intensity to cause intermittency than the fluctuations with higher correlation. These observations would be discussed in detail in the full paper.

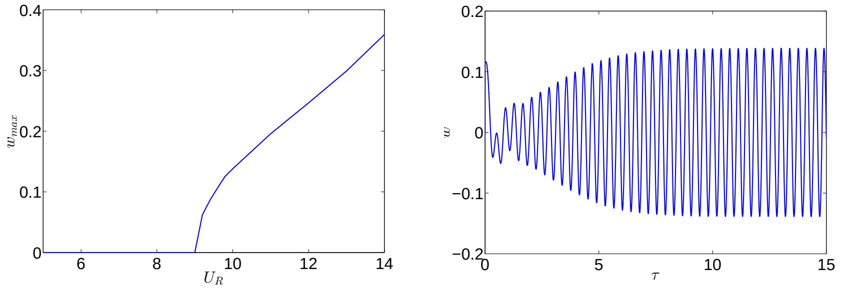

Figure 1: (a) Bifurcation Diagram for the aeroelastic system; (b) Time history for $U_R = 10$.

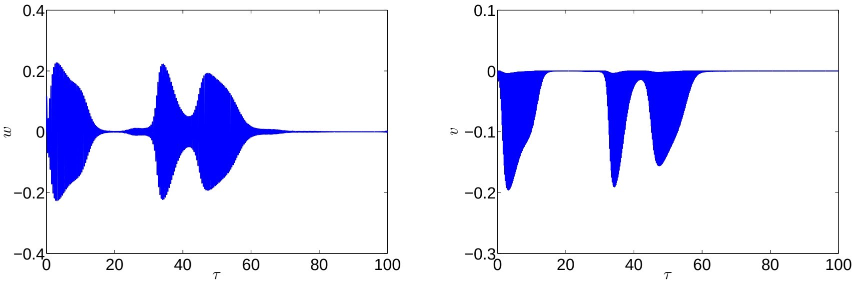

Figure 2: Intermittent responses (a) in the transverse displacement $w(\tau)$ and (b) in the longitudinal displacement $v(\tau)$ when the flow fluctuates about $U_R = 8$ with $\sigma = 0.5$ and $c = 0.1$.

References

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