

Parametric resonance of a nonlinear energy harvester for torsional vibrations

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Summary. Torsional vibrations occur commonly in power transmission systems and components. Often these vibrations are subject to transient frequency response, which are nevertheless dominated by fixed multiples of the main shaft speed (such as engine order vibrations). In this paper, a lightweight electromagnetic oscillator is proposed as an energy harvester from these torsional vibrations. A nonlinear spring is applied to tune the response of the oscillator to parametric excitations. Numerical analysis is carried out to predict output power of the harvester over a wide range of frequencies. The dynamics of the system offers the potential of increasing the effective operation band of the harvester, employing parametric resonance.

Introduction

Harvesting energy from mechanical vibrations is an emerging field pointing to the use of lightweight oscillatory attachments in order to capture energy from a system's secondary vibratory modes [1]. One example is the resident vibration on transmission input shaft, flywheel and the crankshaft in the form of higher harmonics of the engine speed (commonly referred to as engine order vibrations [2]). A major drawback in the current state-of-the-art is the requirement for precise tuning of any vibration absorbing systems such as dual mass flywheel [3] or indeed any linear harvesting device. Multi-modal harvesters, with incorporated non-linearity are efficient tunable oscillators over an extended range of response frequencies. Nonlinear bi-stable oscillators have recently attracted significant attention, mostly due to the possibility of inter-well oscillations [4]. However, this mode of oscillations requires a minimum input energy.

One of the most efficient ways of pumping energy into a system is to subject it to parametric resonance. It is known that linear parametrically-excited systems may experience instability when the excitation frequency is close to twice the natural frequency [5]. Nevertheless, the addition of nonlinear stiffness elements can confine unstable solutions to limit cycles [5]. Hitherto, Parametric vibrations have been considered as amplification mechanisms for regular externally excited systems [6]. This paper investigates the response of an energy harvester to parametric external excitation, focusing primarily on rotational vibratory systems. The aim is to demonstrate that parametric vibrations may allow for dramatically broader frequency band-width in efficient vibration energy harvesting.

System equations

The dynamics of a mechanical oscillator undergoing external and parametric excitation can be described as:

$$\begin{aligned} \ddot{x} + 2\zeta\omega_n\dot{x} + (\omega_n^2 - \Omega_{sh}^2)x + \beta x^3 - \theta I/m &= -g \cos(\Omega_{sh}t) \\ L\dot{I} + (R_i + R_L)I + \theta\dot{x} &= 0 \end{aligned} \quad (1)$$

where x is the oscillator's radial displacement, ζ is the damping ratio, $\omega_n = \sqrt{k/m}$ is the linearised system natural frequency, $\beta = k_3/m$ is the nonlinear stiffness coefficient, m is the mass of the oscillator, Ω_{sh} is the external excitation frequency, representing the speed of a shaft and g is the gravitational acceleration. The magnetic properties of the oscillator establish a coupling with an adjacent electric circuit comprising a coil and a resistive load. For the electric circuit, I denotes the current, L the coil's inductance, R_i the coil's internal resistance, R_L the resistive load and θ the electromagnetic coupling between the coil and an oscillating magnet. Considering slow variations of the excitation frequency so that $\Omega_{sh} = \Omega(1 + \alpha \cos \omega t)$ with $a \ll 1$, Eq. (1) becomes:

$$\begin{aligned} \ddot{x} + 2\zeta\omega_n\dot{x} + (\omega_n^2 - \Omega^2 - 2a\Omega^2 \cos \omega t)x + \beta x^3 - \theta I/m &= -g \cos \Omega t \\ L\dot{I} + (R_i + R_L)I + \theta\dot{x} &= 0 \end{aligned} \quad (2)$$

Eq. (2) shows that the system undergoes a combination of external and parametric excitations. Furthermore, the linearised frequency varies with Ω , causing the system to bifurcate from a mono-stable to a bi-stable response when $\Omega = \omega_n$. This becomes evident if one examines the potential energy of the conservative system. In fact, Fig. (1) shows the normalized potential energy of the system with $\omega_n = 87.23$ rad/s and two values for $\Omega = 62.8$ rad/s and $\Omega = 439.82$ rad/s respectively. It is clear that Eq. (2) incorporates both mono- and bi-stable responses, controlled by the relation between the main shaft speed Ω and the linear frequency ω_n .

As Ω increases, it would dominate the linear part of the stiffness characteristics, so that ω_n may be neglected. In this case, the response of the system to parametric excitation would depend on the ratio of the parametric vibration frequency over the external applied frequency, i.e. ω/Ω . However, in most rotational mechanisms torsional vibrations attain a frequency which is a multiple of the main shaft speed as a result of the design of the system (e.g. x2, x4, x8). Hence, the frequency ratio of the system in Eq. (2) approaches a fixed value, as Ω increases. (see Fig (1b)). Therefore, the system design can be easily adjusted so that it resonates parametrically for a broad range of frequencies.

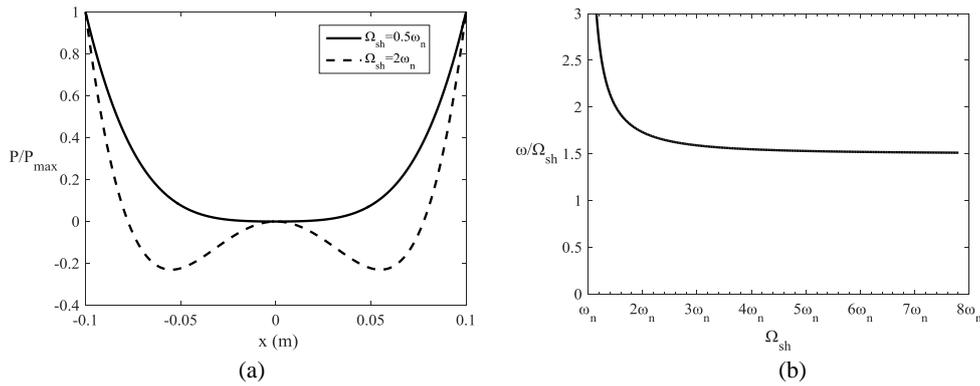


Fig 1. (a) Normalized potential energy of the conservative system corresponding to Eq. (2). Two values of Ω are shown that result to single-well and double-well structure; (b) Frequency of torsional vibrations over the system's linear frequency. As the shaft speed increases, it approaches a fixed value of 1.5.

Numerical Results

The response of the system described by Eq. (2) is computed numerically for a case study in order to demonstrate the resonating response and the consequent broadness of the frequency range of the efficient operation of a harvester. Two coils, sufficiently far from each other, are considered so that the magnet is coupled only to one coil at any given time. Fig. (2a) shows the velocity variation of the oscillator, \dot{x} , with the shaft speed. Two types of response are observed, corresponding to the external part of the excitation, resulting in a vibrating response of the same frequency (1:1, dashed line), and a parametric excitation (1.5:1, solid line). It is clear that the former is confined to a narrow band, when compared with the latter, which initiates shortly after bifurcation to a bi-stable dynamic response. The frequency ratio (see Fig. (1b)) rapidly approaches the primary parametric resonance (around $\omega/\Omega = 2$). This results in sustained vibrations over the remaining frequency range. Consequently, a broadband vibration energy harvesting, is shown in Fig (2b).

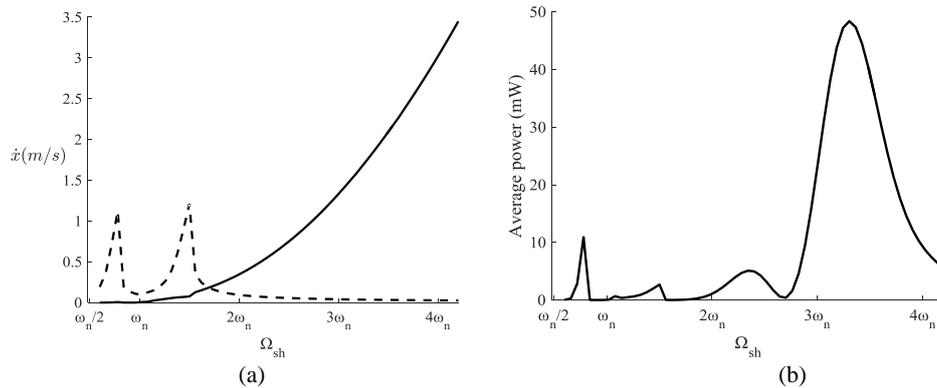


Fig 2. (a) Numerically computed velocity of the oscillator in Eq. (2). (b) Corresponding load power.

Conclusions

The dynamics of an electromagnetic nonlinear energy harvester for rotational mechanisms is investigated. Parametric resonance is utilized to sustain the oscillations of a magnet over a broad range of frequencies. This is based upon the dependence of vibration response frequency superimposed upon the main shaft nominal steady speed. Numerical simulations demonstrate the efficacy of the concept with sustained harvesting of vibration energy and without the need for any system tuning.

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