Advantages of Alpha-Stable Distribution fits for Dynamic Responses of Nonlinear Structures subjected to Random Excitations

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Summary. Dynamic responses of nonlinear systems subjected to random excitations can be important for a range of mechanical and civil systems or structures. The distribution of such dynamic responses can be related to the design of such systems or for deciding intervention options. A good estimate of extreme values of the responses due to random excitation is important since the design, safety, rehabilitation or intervention decisions on a structure is often related to the tail of such extreme value estimates. This paper demonstrates that standard extreme value distributions are often inadequate for characterising the extreme value of dynamic responses of nonlinear systems subjected to random excitations and an alpha-stable distribution framework, when adopted, can give a better representation of such extreme values. The random excitations comprise of white noise, coloured noise and excitations that are represented with alpha-stable distributions. Systems with cubic nonlinearity, bilinear stiffness and with coulomb friction are considered.

Introduction

Stable distribution is important for understanding and modelling measured information (Tsionas, 2016), especially for processes with relatively fat tails (Magdziarz, 2009). In this regard, alpha-stable distributions have been applied to a range of data (Boubchir and Fadili, 2006; Salas-Gonzalez et al., 2009) over traditional distributions. These distributions are often general cases of Ornstein-Uhlenbeck processes (Jakubowski, 2007) exhibiting both Noah and Joseph effects (Magdziarz, 2008). While the potential of applying alpha-stable distributions to practical applications is being acknowledged (Yu et al., 2013), there is not enough evidence-base for such applications for a range of different reasons.

Estimates of extreme values of dynamic responses of structures are important for their design and robustness against risk over lifetime. Loadings are expressed probabilistically for these structures and decisions for safety, rehabilitation or intervention are taken based on responses against a certain percentile of excitation distribution (Quilligan et al., 2012). The distributions are obtained by assuming a reasonable or well-established type of distribution or using data-fits when observations are available. For a wide range of engineering applications, the assessment of extreme values relate to approximating observed or modelled responses of structures at their tails of distributions reasonably to asymptotic distributions like the Gumbel, Weibull or Frechet distributions which are essentially a special case of Generalised Extreme Value (GEV) distribution (Castillo, 2012).

While GEV fits are often reasonable for linear systems, for most natural random excitations and for nonlinear system such an extreme value characterization is not appropriate. An alpha-stable framework has its advantages for such purposes.

Methodology

A random variable X is follows stable distribution with parameters $\vec{\theta}=(\alpha,\beta,\gamma,\delta)$ if the characteristic function takes the form

$$\Phi(u)=E\left[\exp(iuX)\right]=\begin{cases}\exp\left\{-\gamma^{\alpha}|u|^{\alpha}\left[1-i\beta\left(\tan\dfrac{\pi\alpha}{2}\right)(\operatorname{sign}u)\right]+i\delta u\right\} & \alpha\neq 1\\ \exp\left\{-\gamma|u|\left[1+i\beta\dfrac{2}{\pi}(\operatorname{sign}u)(\ln|u|)\right]+i\delta u\right\} & \alpha=1\end{cases} \tag{1}$$

The cumulative distribution function (CDF) of alpha-stable random variable X having parameters $\vec{\theta}=(\alpha,\beta,\gamma,\delta)$ can be expressed as $F_X(x|\alpha,\beta,\gamma,\delta)$. The CDF of alpha-stable distribution does not have an explicit form (except when α take values 1, 1.5 or 2). Hence, the cdf can be estimated by numerically integrating the characteristic function $\Phi(u)$, $F_X(x|\alpha,\beta,\gamma,\delta)=\dfrac{1}{2\pi}\int_{-\infty}^{\infty}\dfrac{e^{-iux}}{iu}\Phi(u)\,du$

Numerical Experiments

We simulate a single-degree-of-freedom (SDOF) system with cubic nonleariry, a bilinear system and one with Coulomb friction with random excitation characterised by distributions appropriate and not appropriate for a GEV representation respectively. We estimate the dynamic response distribution using an alpha-stable approach with the parameters estimated with maximum likelihood, quantile based estimation, empirical characteristic function based estimation, fractional momemnt base estimation, Log absolute moment based estimation and U-Statistic based estimation.

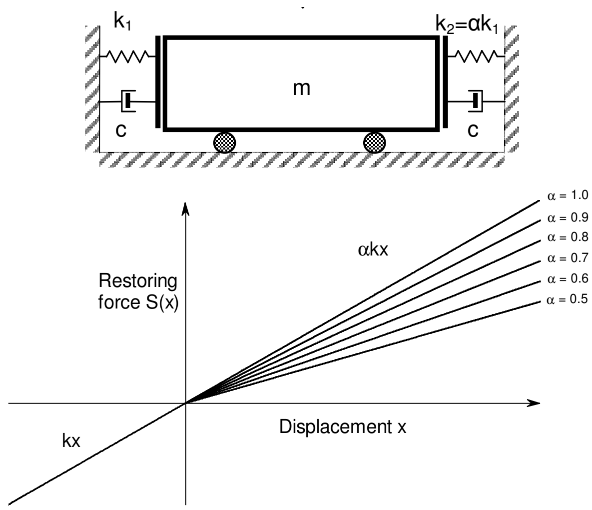

Figure 1. A Single Degree of Freedom System with Bilinear Stiffness

Results

We show that the alpha-stable approach is more appropriate than the GEV approach for modelling dynamic response distributions, especially for nonlinear systems. We also compare how different methods of estimation of paramters of alpha-stable distributions and how the dynamical systems influence the parameters.

Conclusions

An alpha-stable distribution based modelling of dynamical responses of nonlinear systems with random excitations can be more advantageous than traditional GEV fits. Different methods of paramter estimation are calibrated against different nonlinear systems and excitations.

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