Energy Harvesting from Vortex Induced Vibration using Period-1 Rotation of Parametric Pendulum

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Summary. We have studied the possibility of energy harvesting from the vortex induced vibration using period-1 rotating solution of parametric pendulum. For this purpose, a pendulum is installed in such a way that vibration of cylinder acts as base excitation for the pendulum. The coupled equations of motion of the this system are obtained. For some sets of parameters, the existence of period-1 rotation of pendulum is confirmed for this system. But this period-1 rotation does not exist for all initial conditions. So, for continuous energy extraction, we also propose a control to initiate and maintain period-1 rotation from all initial conditions. To study this control in a systematic way, we neglect the effect of pendulum rotation on the vibration of cylinder.

Equations of Motion

Wiercigroch [1] introduced the idea of using rotating solutions of parametric pendulum for energy harvesting from sea waves. Here in this work, we have applied this idea to harvest energy from vortex induced vibration (VIV). Two physical models to extract energy from vortex induced vibration using pendulum are shown in figure 1. The non-dimensional equations of motion for vertical case are

$$\text{Cylinder Oscillator: [2]} \quad \ddot{y} + \left(2\zeta_v\delta_v + \frac{\gamma_v}{\mu_1}\right)\dot{y} + \delta^2 y + \frac{\mu_2}{l_d}(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) = M_v q_w, \quad (1)$$

$$\text{Wake Oscillator: [2]} \quad \ddot{q}_w + \epsilon({q_w}^2 - 1)\dot{q}_w + q_w = A_v\dot{y} \ \text{ and} \quad (2)$$

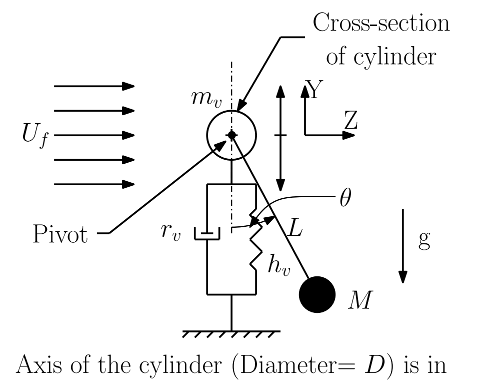
$$\text{Pendulum Equation:} \quad \ddot{\theta} + c_v\dot{\theta} + (a_v + l_d\ddot{y})\sin\theta = 0. \quad (3)$$

The non-dimensional equations of motion for horizontal case are

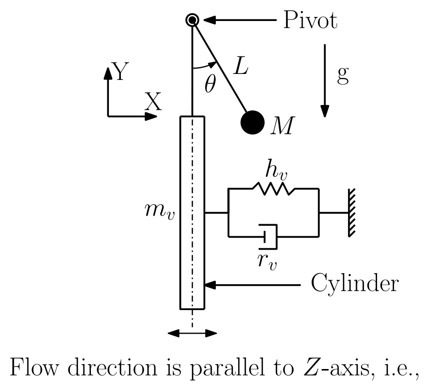
$$\text{Cylinder Oscillator: [2]} \quad \ddot{x} + \left(2\zeta_v\delta_v + \frac{\gamma_v}{\mu_1}\right)\dot{x} + \delta^2 x + \frac{\mu_2}{l_d}(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) = M_v q_w, \quad (4)$$

$$\text{Wake Oscillator: [2]} \quad \ddot{q}_w + \epsilon({q_w}^2 - 1)\dot{q}_w + q_w = A_v\dot{x} \ \text{ and} \quad (5)$$

$$\text{Pendulum Equation:} \quad \ddot{\theta} + c_v\dot{\theta} + a_v\sin\theta + l_d\ddot{x}\cos\theta = 0. \quad (6)$$



(a)



(b)

Figure 1: Physical models of energy extraction from vortex induced vibration using parametric pendulums. (a) Using vertically excited pendulum. (b) Using vertically excited pendulum. As this is 2-D model, the length of the cylinder is taken to be 1 unit.

Descriptions of System Variables and Parameters:

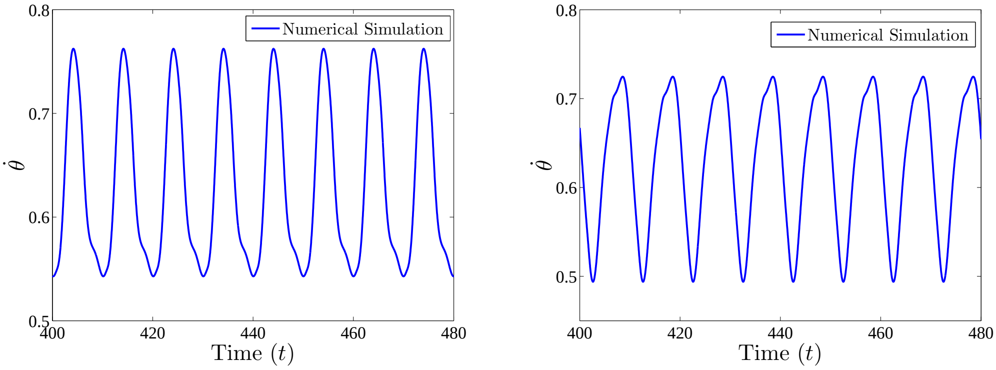
$y = Y/D$ and $x = X/D$, where Y, X = displacements of cylinder in the Y (vertical) and X (horizontal) directions respectively. $q_w = 2C_L/C_{L0}$, where C_L =vortex lift coefficient and C_{L0} = vortex lift coefficient on a fixed structure subjected to vortex shedding.

$m_v = m_s + m_f$, where m_s = mass of cylinder, $m_f = C_M\rho\pi D^2/4$, ρ = density of fluid and C_M = added mass coefficient. $r_v = r_s + r_f$, where r_s =damping in cylinder structure, $r_f = \gamma_v\Omega_f\rho D^2$, γ_v= stall parameter (assumed to be constant), $\Omega_f = 2\pi StU_f/D$, St =Strouhal number $= D\Omega_f/(2\pi U_f)$.

$\Omega_s = \sqrt{h/(m_v + M)}$, $\delta_v = \Omega_s/\Omega_f$, $\zeta = r_s/[2(m_v + M)\Omega_s]$, $\mu_1 = (m_v + M)/(\rho D^2)$, $l_d = D/L$, $\mu_2 = M/(m_v + M)$, $M_v = C_{L0}/(16\pi^2 St^2 \mu_1)$, ϵ = small bookkeeping parameter, A_v = velocity coupling coefficient from cylinder to wake, $c_v = C/(\Omega_f M L^2)$, C = damping corresponding to at pendulum motion at pivot and $a_v = g/(L\Omega_f^2)$.

Existence of Period-1 Rotation

For certain values of parameters, period-1 rotation exists for these two systems, see figure 2. This period-1 rotation can be exploited to harvest energy from vortex induced vibration. But this period-1 rotation coexists with $(\theta, \dot{\theta}) = (0, 0)$ and oscillation in case of vertical excitation and with some other oscillation in case of horizontal excitation. So whole $\left(\theta(0), \dot{\theta}(0)\right)$ space does not belong to the basin of attraction of period-1 rotation. Thus an external control torque is



(a) (b)

Figure 2: Period-1 anticlockwise rotation. (a) Vertical case, i.e., Eqs. (1), (2) and (3). (b) Horizontal Case, i.e., Eqs. (4), (5) and (6). Set of parameters: $\zeta_v = 0.02, \delta_v = 1, \gamma_v = 0.8, \mu_1 = 10, \mu_2 = 0.04, l_d = 0.1, M_v = 0.118, c_v = 0.03, a_v = 0.06, \epsilon = 0.3$, and $A_v = 12$.

needed to initiate and maintain period-1 rotation from all initial conditions for continuous energy extraction.

Neglecting the Effect of Pendulum on Cylinder

To propose and study a feedback control in a systematic way, we neglect the effect of pendulum rotation on the vibration of cylinder, i.e., we put $\mu_2 = 0$. Also for very small ϵ, the vibration of the cylinder becomes harmonic. For this limiting case, the two current systems with an external control torque can be studied using the following equations:

$$\text{Vertical:} \quad \ddot{\theta} + c\dot{\theta} + (a - b\cos t)\sin\theta = F_c \quad \text{and} \tag{7}$$

$$\text{Horizontal:} \quad \ddot{\theta} + c\dot{\theta} + a\sin\theta - b\cos t\cos\theta = F_c, \tag{8}$$

where F_c = external control torque. The study regarding control torque using the Eq. (7), i.e., for vertical case can be found the reference [3]. This control torque is inspired from the delayed control of Pyragas [4].
For $\mu_2 \neq 0$ and ϵ not so small, the vibration of cylinder no longer remains harmonic. Thus for the same set of pendulum parameters, the basin of attraction of period-1 rotation becomes very much smaller than previous. In this work, we shall show that this proposed control torque works well when $\mu_2 \neq 0$ and ϵ not so small for both the physical models.

References

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