2D Control of Energy Transport in the Locally Resonant Unit Cell Model with Self Excitation

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Summary. Present work is devoted to the analysis of resonant energy transport emerging in the strongly nonlinear, locally resonant, 2D unit-cell model. The system under consideration comprises an outer mass with the internal inclusion (e.g. rotator, 2D vibro-impact internal inclusion, 2D nonlinear oscillator) subject to the 2D, external self-excitation. In the current study, we revealed the emergence and bifurcations of highly nonlinear, nonstationary regimes manifested by the unidirectional energy localization as well as the complete, bidirectional energy transport controlled by the internal inclusion. We show that bifurcations of these highly nonstationary regimes can be analysed on the phase plane using the singular asymptotic analysis.

In the present work we aim at studying the limit of high energy excitations leading to the resonant, recurrent, and complete energy transport between the axial and the lateral vibrations of the outer element as well as the spontaneous, unidirectional energy localization controlled by the motion of the internal inclusion.

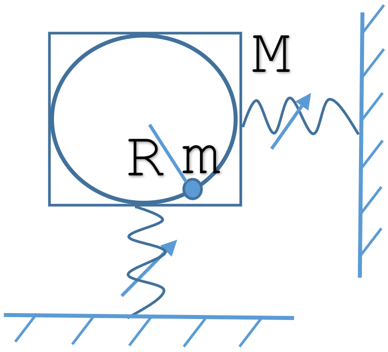

Figure 1 Scheme of the system under consideration: outer mass containing rotator is subject to the nonlinear local potential and self excitation in the two orthogonal directions (i.e. axial and lateral).

As an example we illustrate the model comprising an outer mass with internal rotator subject to the 2D nonlinear local potential and self-excitation assumed in the axial and lateral directions. In the present study we make the following assumptions: (1) the motion of the system is in-plane, (2) gravity is not taken into account. The equations of motion read the following:

$$\begin{aligned}&(M+m)\frac{d^2x_1}{dt^2}+kx_1+Kx_1^3-mR\left(\ddot{\theta}\sin\theta+\dot{\theta}^2\cos\theta\right)+(\kappa_1-\kappa_2x_1^{\,2}+\kappa_3x_1^{\,4})\frac{dx_1}{dt}=0;\\&(M+m)\frac{d^2x_2}{dt^2}+kx_2+Kx_2^3+mR\left(\ddot{\theta}\cos\theta+\dot{\theta}^2\sin\theta\right)+(\kappa_1-\kappa_2x_2^{\,2}+\kappa_3x_2^{\,4})\frac{dx_2}{dt}=0;\\&mR\frac{d^2x_2}{dt^2}\cos\theta-mR\frac{d^2x_1}{dt^2}\sin\theta+mR^2\ddot{\theta}=0.\end{aligned} \tag{1}$$

In the present work, we study the resonant interaction (i.e., 1:1:1 resonance) between the axial and lateral motions of the outer mass controlled by the motion of the internal inclusion. Using this assumption, we transform the system to the dimensionless form:

$$\begin{aligned}&\xi_1''+k\xi_1-\varepsilon\left(\theta''\sin\theta+\theta'^2\cos\theta\right)+\varepsilon(k_1-k_2\xi_1^{\,2}+k_3\xi_1^{\,4})\xi_1'+\varepsilon\alpha\xi_1^{\,3}=0;\\&\xi_2''+k\xi_2+\varepsilon\left(\theta''\cos\theta-\theta'^2\sin\theta\right)+\varepsilon(k_1-k_2\xi_2^{\,2}+k_3\xi_2^{\,4})\xi_2'+\varepsilon\alpha\xi_2^{\,3}=0;\\&\xi_2''\cos\theta-\xi_1''\sin\theta+\theta''=0;\end{aligned} \tag{2}$$

Further, introducing the complex variables: $\xi_k'=\frac{1}{2}\left(\psi_k+\psi_k^{\,*}\right)$; $\xi_k=-\frac{i}{2}\left(\psi_k-\psi_k^{\,*}\right)$ and implementing an asymptotic procedure separating the slowly varying complex amplitudes of the axial and the lateral motions as well as the fast oscillations: $\psi_k=\varphi_k(\tau)\,\mathrm{e}^{i\tau};k=1,2;\theta=\tau+\beta(\tau)$; $\frac{d}{d\tau}=\frac{\partial}{\partial\tau_1}+\varepsilon\frac{\partial}{\partial\tau_2};\varphi_k=\varphi_{k0}(\tau_1,\tau_2)+\mathrm{O}(\varepsilon)$; and suggest $\beta(\tau)=B(\tau_2)+\mathrm{O}(\varepsilon)$.

To study the evolution of the system in the vicinity of 1:1:1 resonance surface, using singular multi-scale analysis, we reduce consideration to the evolution of the averaged flow as a whole on the two (stable and unstable) slow invariant manifolds.

$$\frac{\partial \varphi_{k0}}{\partial \tau_2} = i\frac{3\alpha}{8}|\varphi_{k0}|^2\varphi_{k0} + \frac{(-i)^{k-1}}{2}e^{iB} - \left(\frac{k_1}{2}\varphi_{k0} - \frac{k_2}{4}|\varphi_{k0}|^2\varphi_{k0} + \frac{k_3}{16}|\varphi_{k0}|^4\varphi_{k0}\right)\Big] = 0; \quad (3)$$
$$-\frac{1}{4}\left[\left(\varphi_{10}{}^{*}e^{iB} - \varphi_{10}e^{-iB}\right) - i\left(\varphi_{20}{}^{*}e^{iB} - \varphi_{20}e^{-iB}\right)\right] = 0.$$

This slowly evolved system depicts the dynamics of the full system in the vicinity of the fundamental resonance (1:1:1). On each of the two slow invariant manifolds the internal inclusion of the unit-cell plays a role of the effective coupling between the two motions of the outer mass element, namely resonant energy flow from axial to lateral vibrations of the outer unit.

Motion of the rotator dictates the regimes of energy transport between the axial and lateral vibrations reduced on the two separate phase planes. At each plane one can introduce a system in the angular variables: $\varphi_{10} = N\cos\theta\, e^{i\delta_1}$; $\varphi_{20} = N\sin\theta\, e^{i\delta_2}$; $\Delta = \delta_1 - \delta_2$ which is described by the following set of equations,

$$\frac{d\theta}{d\tau_2} = (-1)^m \frac{\cos\Delta}{2\sqrt{N}\sqrt{1+\sin\Delta\sin 2\theta}} - d\sin 4\theta; \quad (4)$$
$$\frac{d\Delta}{d\tau_2} = \sigma\cos 2\theta - (-1)^m \frac{\cos 2\theta \sin\Delta}{\sqrt{N}\sin 2\theta\sqrt{1+\sin\Delta\sin 2\theta}};$$

where $\sigma = 3\alpha N^3/16$ - characterizes conservative nonlinearity, $d = k_1/8$ - dissipative terms. Considering the reduced system, we performed analysis on the phase plane which allows to formulate the analytical criterion for the formation of intensive energy transport as well as the transition to unidirectional energy locking. In Figure 2 we illustrate the two phase planes of (4) corresponding to the regimes of intense energy flow.

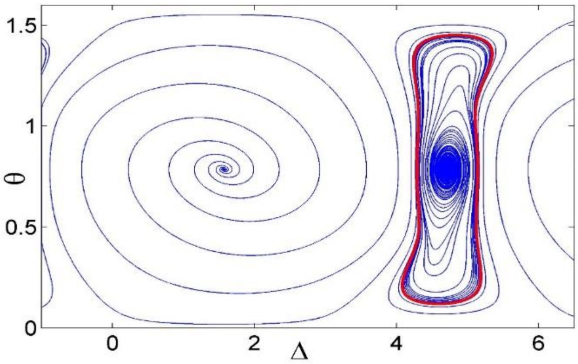

Figure 2 Stable Limit Cycle (denoted with the red color) demonstrating intensive energy transport between the axial and the lateral vibrations of the outer element, System parameters: d=0.6, σ=0.6.

Conclusions

Dynamic response of the locally resonant, 2D unit cell model to high energy self-excitation is considered analytically and numerically. Special analytical treatment based on the direct complexification averaging as well as the singular multi-scale analysis of the averaged flow is developed. We study the existence and bifurcations of highly nonstationary regimes of massive energy transport as well as the regime of unidirectional energy locking emerging in the locally resonant model under consideration. The analysis is performed by the reduction of the global flow on the resonant invariant manifold (SIM) in the neighbourhood of the fundamental resonance (1:1:1).

References

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