Inertia Properties and Their Role in Haptic Rendering

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Summary. In this paper we illustrate how the effective inertia perceived by the human operator can be best represented with mathematical models that allows one to determine optimum designs and operating scenarios for various haptic interfacing problems. The particular problem we investigate is that how the effective inertia properties can be shaped and controlled to influence the quality and performance of haptic interaction when the rendering work space has lower dimensions than the number of actuated degrees of freedom of the device. A model-based control method is applied for increasing the effective inertia, and an experimental study is used to validate this method by measuring the impedance range of a commercial haptic device.

Introduction

Force feedback is a primary means to develop haptic systems. An important element of such haptic systems is the haptic device that forms the link between the human operator and the virtual environment representation. The mechanical properties and dynamics of the device can be a significant factor in shaping the operation and performance of the haptic system. Device dynamics and inertia properties can substantially contribute in both high- and low-impedance rendering.
From the perspective of the human operator the mechanical design and the mechanics of the device is primarily manifested in terms of the inertia properties perceived by the operator. These effective inertia properties depend on the general mechanical design of the system, which includes, the geometry of the links, the kinematic pairs connecting the links, and the kinetic properties of the links such as mass, centre of mass, and the inertia tensor [1, 2]. Little is known about the role of these effective inertia properties in haptic rendering. It is often an objective to reduce the mass and inertia of the device to as small values as possible to increase transparency. But, this is not necessarily the best strategy. For example, it can be shown that devices with very small perceived mass will have a quite limited range of operation for the case of high-impedance interactions. More analysis is needed to understand how the effective inertia properties really contribute to the perception and the dynamics of haptic interactions. Here, we show that how to increase the effective inertia by control in order to achieve higher virtual impedances, and in turn a more realistic haptic rendering of stiff virtual objects.

Effective Inertia Representation

It can be shown that the dynamic model of a haptic system can be written in the form

$$\mathbf{W}_{rr}\dot{\mathbf{u}}_r + \mathbf{W}_{ra}\dot{\boldsymbol{\pi}}_a + \mathbf{z}_r = \mathbf{f}_r \tag{1a}$$

$$\mathbf{W}_{ar}\dot{\mathbf{u}}_r + \mathbf{W}_{aa}\dot{\boldsymbol{\pi}}_a + \mathbf{z}_a = \mathbf{f}_a \tag{1b}$$

where $\mathbf{W}_{rr}$, $\mathbf{W}_{ra}$, $\mathbf{W}_{ar}$ and $\mathbf{W}_{aa}$ are the blocks of the mass matrix corresponding to the parameterization $\mathbf{u}_r = \mathbf{J}_r\dot{\mathbf{q}}$ and $\boldsymbol{\pi}_a = \mathbf{J}_a\dot{\mathbf{q}}$. The generalized velocities $\mathbf{u}_r$ and $\boldsymbol{\pi}_a$ are representing the rendered and admissible motions, respectively, $\mathbf{J}_r$ and $\mathbf{J}_a$ are the corresponding Jacobians, and $\dot{\mathbf{q}}$ is the array of joint velocities. In addition, $\mathbf{z}_r$ and $\mathbf{z}_a$ are the Coriolis and centrifugal terms, and $\boldsymbol{\tau} = \mathbf{J}_r^{\mathrm{T}}\mathbf{f}_r + \mathbf{J}_a^{\mathrm{T}}\mathbf{f}_a$ is the joint torque with $\mathbf{f}_r$ and $\mathbf{f}_a$ denoting the generalized applied forces.
The effective inertia associated with the rendered workspace can be derived in the form $\mathbf{W}_{\text{eff}} = \mathbf{W}_{rr} - \mathbf{W}_{ra}\mathbf{W}_{aa}^{-1}\mathbf{W}_{ar}$ [2]. This quantity significantly influences the dynamic behaviour of the system. It is a tensorial quantity which reduces to a scalar for the case of single-dimensional rendering, e.g., normal interaction with a virtual surface. It can be shown that in high-impedance rendering the effective inertia has a stabilizing effect in general. In low-impedance cases, the human operator has larger movements and the effective inertia and its coupling to the other degrees of freedom of the device can have an important effect on the human perception.

Shaping of the Dynamic Behaviour

The formulation above reflects that the coupling between the rendered workspace and the other redundant degrees of freedom can have a significant effect, which can be exploited by mechanical design or control to influence the performance of the system. The effect of coupling on the effective inertia is represented by the $\mathbf{W}_{ra}\mathbf{W}_{aa}^{-1}\mathbf{W}_{ar}$ term. This can be readily seen by expressing $\dot{\boldsymbol{\pi}}_a$ from (1b) and substituting it into (1a) which result in

$$\left(\mathbf{W}_{rr} - \mathbf{W}_{ra}\mathbf{W}_{aa}^{-1}\mathbf{W}_{ar}\right)\dot{\mathbf{u}}_r = \mathbf{f}_r - \mathbf{W}_{ra}\mathbf{W}_{aa}^{-1}\left(\mathbf{f}_a - \mathbf{z}_a\right) - \mathbf{z}_r \tag{2}$$

where the coefficient matrix of $\dot{\mathbf{u}}_r$ gives also an intuitive definition for the effective inertia. Matrix $\mathbf{W}_{aa}$ is always positive definite, hence, the coupling term $\mathbf{W}_{ra}\mathbf{W}_{aa}^{-1}\mathbf{W}_{ar}$ always reduces the effective inertia. This can be influenced by the mechanical design of the system or its reconfiguration using redundancy as the terms in the expression of $\mathbf{W}_{\text{eff}}$ are all linked to the physical dimensions, states and parameters.
On the other hand, using the dynamic equations in (1) it is also possible to shape the effective inertia via the appropriate selection of the applied forces acting in the admissible directions. To remove the coupling, $\mathbf{f}_a$ has to be selected such that

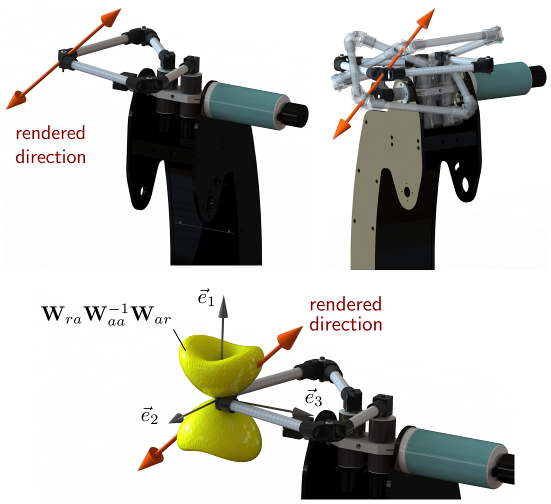

Figure 1: Haptic device and its motion in the rendered direction (top), Representation of coupling in the operational space (bottom)

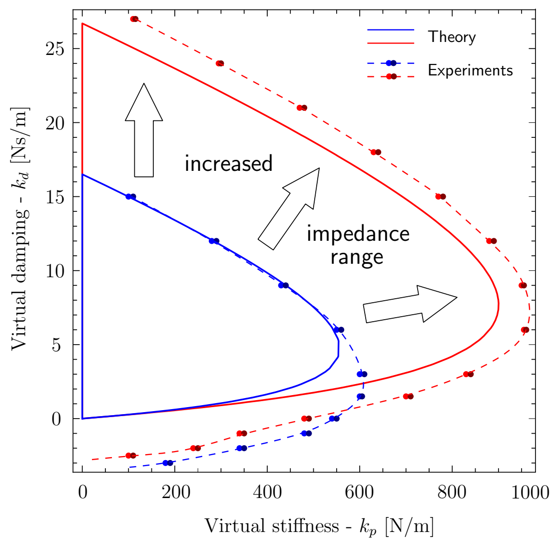

Figure 2: Theoretical and experimental impedance ranges

$\dot{\boldsymbol{\pi}}_a = \mathbf{0}$ which condition can be analyzed by eliminating $\dot{\mathbf{u}}_r$ from (1b). This results in

$$\left(\mathbf{W}_{aa} - \mathbf{W}_{ar}\mathbf{W}_{rr}^{-1}\mathbf{W}_{ra}\right)\dot{\boldsymbol{\pi}}_a = \mathbf{f}_a - \mathbf{W}_{ar}\mathbf{W}_{rr}^{-1}\left(\mathbf{f}_r - \mathbf{z}_r\right) - \mathbf{z}_a \tag{3}$$

from which the necessary feed-forward control term that minimizes the coupling becomes

$$\mathbf{f}_a = \mathbf{W}_{ar}\mathbf{W}_{rr}^{-1}\left(\mathbf{f}_r - \mathbf{z}_r\right) + \mathbf{z}_a \tag{4}$$

It can be shown that in this case the effective inertia is maximized with respect to a certain rendered direction and the configuration of the system, which is generally advantageous for high-impedance rendering. Also, this action minimizes the effect of perturbations possible in the redundant degrees-of-freedom represented by $\boldsymbol{\pi}_a$. This can significantly increase the feeling of realism in rendering.

Below we illustrate this experimentally for high impedance rendering using the haptic mechanism shown in Fig. 1. We consider the uncoupled system, the mechanism alone, and the interaction with a virtual surface with normal direction indicated by the red arrows. The complex motion of the five-link mechanism of the device along this direction is magnified and illustrated also in Fig. 1. Since only a single rendered direction is considered, the coupling inertia term will be scalar and can also be visualized in the workspace of the device as a parametric surface. This is shown in Fig. 1, together with the eigenvectors $\vec{e}_i$, $i = 1, 2, 3$ of the mass matrix. The direction of maximum coupling is in the plane determined by $\vec{e}_1$ and $\vec{e}_3$ which correspond to the largest and smallest eigenvalues. In the case of the experimental device and configuration, Fig. 1 indicates significant coupling associated with the rendered direction. This results in poor stability and transparency properties which can considerably be improved by applying the feed-forward control term from Eq. (4).

The stability of the haptic task illustrated in Fig. 1 was analyzed by considering the linearized system of dynamic equations obtained from (1) and interpreting the mass matrix for the investigated configuration. The virtual surface force was represented as $\mathbf{f}_r = -k_p\mathbf{x} - k_d\mathbf{v}$ where $\mathbf{x}$ is the local coordinate level representation of the rendered direction and $\mathbf{v}$ is the corresponding velocity obtained through backward differentiation followed by filtering with a 2nd order Butterworth filter having cut-off frequency of 20 Hz . The coefficients k_p and k_d are the applied virtual stiffness and damping factors. In order to avoid the saturation of the direct drive actuators, the experiments were carried out at a reduced, 200 Hz, sampling rate resulting in a smaller impedance range.

The corresponding theoretical and experimental impedance ranges are illustrated in Fig. 2. The area enclosed by the solid blue line represent the calculated stable domain without shaping the effective inertia. Whereas, the considerably larger area enclosed by the solid red boundary shows the stable domain of operation when the effective inertia is maximized by applying the feed-forward control term (4). These results are confirmed by the experimental measurements shown as solid dots with the same colors. The lighter and darker dots of each color indicate the lower and upper error bounds of the repeated experiments.

Conclusions

The experiments show that shaping the effective mass by feed-forward control action, the quality of haptic rendering can significantly be increased without the need of changing the basic haptic rendering algorithm, relocating the task, or modifying the design of a haptic interface. Deviations from the calculated impedance ranges may result from modelling errors and friction effects.

References

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