Non-smooth torus to identify domain of attraction of stable milling processes

Zoltan Dombovari*, Jokin Munoa*,**, Rachel Kuske*** and Gabor Stepan *
* *Department of Applied Mechanics, Budapest University of Technology and Economics, Budapest, Hungary*
** *Dynamics and Control, IK4-Ideko, 20870 Elgoibar, Basque Country, Spain*
*** *School of Mathematics, Georgia Institute of Technology, Atlanta, USA, GA*

Summary. The presented work shows a possible model dealing with the non-smooth flyover effect in milling processes. The excitation force of the general modal model of milling process is delayed, nonlinear, time-periodic and piecewise smooth. Apart from a state independent switch originated from the so-called radial immersion, flyover causes difficulties in connection with accurate depiction of the anyway quasi-periodic solution. From industrial point of view, the 'size' of this non-smooth quasi-periodic solution is essential to predict approximately the attraction zone of the stable time-periodic stationary solution calculated using linear theories.

Introduction

Recent technological trends tend to introduce cyber physical system (CPS, [1]) solutions in machine tools to self-sense and self-act during cutting operations. These capabilities of this envisioned enhanced machine tool [2] need to be strengthened by more accurate modelling of the combined controlled cutting process. To ensure quality and productivity requirements vibration must be attenuated. The regeneration, when the past motion of the tool excites the dynamics via the just cut surface is known since the middle of the last century published by the Pioneers in [3] and in [4]. Mathematically, the system can be represented by delay differential equations (DDEs), which generate infinite dimensional phase space. Particularly, milling is a time periodic delayed system. By this form, the asymptotic stability of the stationary cutting solution can be predicted as an important technological requirement in the industry. Unstable stationary cutting leads to the onset of growing vibration limited by a threshold effect when cutting edges, often irregularly, leave and enter the cutting state. This, generally high amplitude limiting vibration is mathematically stable. In its developed form, it is referred as chatter vibration, while the threshold effect that limits the vibration is called as fly-over by the machine tool industry. Apart from the purely nonlinear origin bistable region caused by smooth quasi-periodic solutions in milling [5], tight attraction zone can be formed around the stable stationary solution due to only the non-smooth fly-over effect. This was first experienced using time-domain solution in the motivational work in [6]. In this article island-like stability domain (figure 1a) were presented that are enclosed by Hopf- and period doubling (PD)-kind stability boundaries. As it was shown in that paper, this was originated from the interactions of multiple modes through modulations of the main vibration frequency of the critical 'self-excited' solution. However, in the presented particular example, this island was experienced to have tight attraction zone (figure 1b) by simulation of the corresponding piecewise linear but fly-over model. This work shows methods to predict the size of the attraction zone approximately and also to envision a general numerical description to calculate the real threshold case as a quasi-periodic solution.

Milling model with fly-over

In order to have an industrially acceptable model multiple modes are considered with their modal coordinates $\mathbf{q}$. The model shown below includes proportionally damped modes with modal damping ratios ξ_k and natural angular frequencies $\omega_{\mathrm{n},k}$ characterised by real-valued mode shapes $\mathbf{U}$. Spatial solution is $\mathbf{r}(t) = \mathbf{U}\mathbf{q}(t)$ ($\mathbf{r} = \mathrm{col}(x, y, z)$), while the state is given by $\mathbf{q}_t(\vartheta) = \mathbf{q}(t+\vartheta)$ if $\vartheta \in [-\tau_{\max}, 0]$. In this case the governing equation and the regenerative force are given as

$$\ddot{\mathbf{q}}(t) + [2\xi_k\omega_{\mathrm{n},k}]\dot{\mathbf{q}}(t) + [\omega_{\mathrm{n},k}^2]\mathbf{q}(t) = \mathbf{U}^{\mathsf{T}}\mathbf{F}(t, \mathbf{r}_t(\vartheta)), \quad \mathbf{F}(t, \mathbf{r}_t(\vartheta)) = -a_{\mathrm{p}}\sum_{i=1}^{Z} g_i(t)\mathbf{T}(\varphi_i(t))\mathbf{f}(g_i(t)h_i(t, \mathbf{r}_t(\vartheta))) \qquad (1)$$

and described completely in [5]. Possible nonliearities are originated from the specific cutting force characteristics $\mathbf{f}(h)$, which carries the regeneration (delay) effect denoted symbolically by $\mathbf{r}_t(\vartheta)$ in momentary chip thickness $h_i(t, \mathbf{r}_t(\vartheta))$. This model is able to describe milling processes with momentary positions $\varphi_i(t)$ of the uniformly placed edges of the rotating milling tool. The asymptotic stability of the time periodic stationary cutting solution $\mathbf{q}_{\mathrm{p}}(t) = \mathbf{q}_{\mathrm{p}}(t+T)$ ($T = 2\pi/\Omega$) can be determined by perturbation $\mathbf{u}(t) = \mathbf{q}(t) - \mathbf{q}_{\mathrm{p}}(t)$ and linearisation of (1) around $\mathbf{q}_{\mathrm{p}}$, which results in the so-called stability lobe diagrams (see figure 1a).

Different discontinuity effects in milling

The switching function in (1) swaps different excitation patterns causing discontinuities in second derivatives of $\mathbf{q}(t)$. It can be separated to a state independent screen function related to the radial immersion g_{ri} and a state dependent fly-over related one g_{fo} as

$$g_i(t) := g_i(t, \mathbf{q}_t(\vartheta)) = g_{\mathrm{ri}}(\varphi_i(t))g_{\mathrm{fo},i}(t, \mathbf{q}_t(\vartheta)). \qquad (2)$$

Considering the state independent radial immersion with entry φ_{en} and exit φ_{ex} angles is straightforward and widely taken into account in piecewise analytical and numerical solutions with $g_{\mathrm{ri}}(\varphi_i(t)) = 1$ if $\varphi_{\mathrm{en}} < (\varphi_i(t) \bmod 2\pi) <$

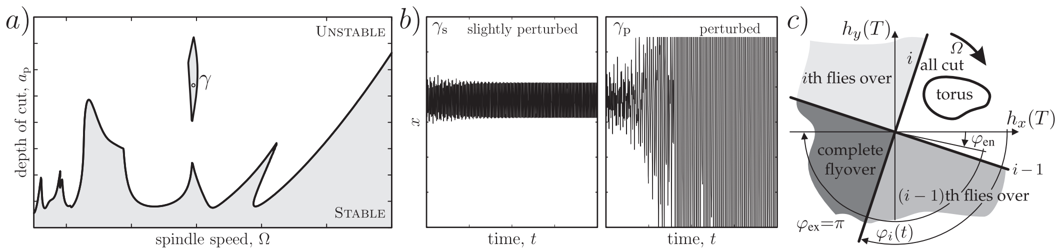

Figure 1: a) stability chart with an island, b) time domain simulations at γ , c) fly-over in milling according to [5].

φ_{ex}, otherwise $g_{\mathrm{ri}}(\varphi_i(t)) = 0$. The time and state dependent switching conditions $h_i(t, \mathbf{q}_t(\theta)) = 0$ (see figure 1c) are causing the real problems in this framework. These are described with $g_{\mathrm{fo},i}(t, \mathbf{q}_t(\vartheta)) = 1$ if $h_i(t, \mathbf{q}_t(\vartheta)) > 0$, otherwise $g_{\mathrm{fo},i}(t, \mathbf{q}_t(\vartheta)) = 0$. Remark that, in required stable stationary solution, when $\lim_{t\to\infty} \mathbf{q}(t) = \mathbf{q}_{\mathrm{p}}(t)$ the $g_{\mathrm{fo},i}$'s are simply not active. Instability on stationary solution or nonlinearity induced unstable tori solutions (bistability, see [5]) 'push' the solution to reach $g_{\mathrm{fo},i}$'s.

A prediction of chattering amplitude

The case presented in figure 1a contains an island with tight attraction zone tested in figure 1b. Accordingly, a method is needed that only serves a rough answer for the size of the attraction zone, and can be used for characterization of 'linearly' stable domain easily using the results of the asymptotic stability analysis of $\mathbf{q}_{\mathrm{p}}(t)$. Time domain based asymptotic methods like semi-discretization (SD) is able to determine the so-called Floquet multipliers μ for the corresponding time periodic system with $T_Z = T/Z$. At the stability border its magnitude is simply $|\mu_{\mathrm{cr}}| = 1$. By using the corresponding eigenvector $\mathbf{s}_{\mathrm{cr}}$, a sample perturbation $\mathbf{u}(\theta) = \mu_{\mathrm{cr}}\mathbf{s}_{\mathrm{cr}}(\theta) + \overline{\mu}_{\mathrm{cr}}\overline{\mathbf{s}}_{\mathrm{cr}}(\theta)$ can be given along the stability boundary that can be used for constructing a prototype of the quasi-periodic solution with frequencies $\omega_{Z,l} = 2\pi/T_Z l$ and $\omega_q = |\arg \mu_{\mathrm{cr}} + 2\pi q|/T_Z$. These latter ones are described in the numerical solution of $\mathbf{s}_{\mathrm{cr}}(\theta)$ originated from the SD. As $\omega_{Z,l}$ and ω_q are not locked, and the correct 'phase shift' is granted by SD between $\mathbf{q}_{\mathrm{p}}$ and $\mathbf{u}$, the quasi-periodic solution densely occupies the invariant torus with $\mathbf{q}_{\mathrm{QP}}(t, r) = \mathbf{q}_{\mathrm{p}}(t) + r\mathbf{u}(t)$. If $\mathbf{f}(h)$ in (1) is linear, and we assume only slight change in spectrum ω_q, the switching conditions can be granted with $h_i(t, \mathbf{q}_{\mathrm{QP}}(t, r)) = 0$ for $i = 1, 2, \ldots, Z$. Then r gives an idea what magnitude the attraction zone can have around the region in question. Of course, in linear case, there is no nonlinearity induced bistability as it is described in [5], but perhaps the encirclement of the island in figure 1a by three different borders creates an only non-smooth bistability region. Apart from predicting roughly the attraction zone, this prediction can be used as a prototype solution for unstable/stable transition for dynamic bifurcation analysis.

Numerical non-smooth boundary solver

For more accurate calculation, a transition between a nonlinear and linear description, Enders' exponential cutting force characteristics can be used. It can be parametrised as $\mathbf{f}(h) := \mathbf{f}_{\mathrm{E}}(h, \gamma) = K_{\mathrm{e}}(1 - \mathrm{e}^{-h/\gamma}) + K_{\mathrm{c}}h$. Here γ is the transition parameter, while K_{e}, and K_{c} are technological parameters. By taking the limit of γ toward zero in $\mathbf{f}_{\mathrm{E}}(h, \gamma)$, the structure of the non-smooth induced bistability can be unfolded over the island depicted in figure 1a. In the end, to reveal the real structure of the dynamics 'around' the island, the nonlinear/linear transition has to be followed by an appropriate boundary value problem (BVP) solver. Calculating an invariant torus even for state independent discontinuity g_{ri} has to be represented with an appropriate mesh. Starting from a critical stationary cutting solution, g_{fo} can be reached along the subcritical branch as in [5]. Passing this first grazing of the solution, the non-smooth condition function(s) g_{fo} has(have) to be represented by irregular triangularisation of the invariant torus by using proper interpolation and quadrature schemes.

Acknowledgement

ERC Advanced grant no. 340889., János Bolyai Research Scholarship (BO/00589/13/6) and OTKA grant no. K108779.

References

[1] Monostori, L., Kadar, B., Bauernhansl, T., Kondoh, S., Kumara, S., Reinhart, G., Sauer, O., Schuh, G., Sihn, W., and Ueda, K., 2016. "Cyber-physical systems in manufacturing". *CIRP Annals - Manufacturing Technology,* ***65***(2), pp. 1–21.

[2] Muñoa, J., Beudaert, X., Erkorkmaz, K., Iglesias, A., Barrios, A., and Zatarain, M., 2015. "Active suppression of structural chatter vibrations using machine drives and accelerometers". *CIRP Annals - Manufacturing Technology,* ***64***(1), pp. 385–388.

[3] Tlusty, J., and Spacek, L., 1954. *Self-excited vibrations on machine tools*. Nakl. CSAV, Prague. in Czech.

[4] Tobias, S., 1965. *Machine-tool Vibration*. Blackie, Glasgow.

[5] Dombovari, Z., and Stepan, G., 2015. "On the bistable zone of milling processes". *Phil. Trans. R. Soc. A,* ***373***(2051), p. 20140409.

[6] Munoa, J., Dombovari, Z., Mancisidor, I., Yang, Y., and Zatarain, M., 2013. "Interaction between multiple modes in milling processes". *Machining Science and Technology,* ***17***(2), pp. 165–180.