Coupled Dry Friction Models in Problems of Aviation Pneumatics'.

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Summary. It is proposed further development of the theory of multi-component dry friction which consists in presenting a more convenient form of the coupled friction models for the problems of the aviation pneumatics dynamics. The procedure of the models constructing consists of the two parts. In the first part, the exact integral expressions for the net vector and torque are formed with used results from the theory of elasticity. In the second part the exact integral models are approximated by smooth analytical functions. The approximate models preserve all analytical properties of the models based on the exact integral expressions. Another one of the improvements of the developed dry friction models consists in the accounting of the real distribution of the contact pressure. The quasi-static contact pressure distribution for the real tire of diagonal structure was obtained using finite element simulation for several load cases. The numerical solution was interpolated by the Legendre polynomials, so that the obtained representation allows one to obtain the coefficients of the approximated dry friction models.

Coupled Dry Friction Models

The dry friction exact integral models in the case of simultaneously sliding, spinning and rolling are constructed for circular contact sites under the assumption that the Coulomb law in differential form holds for the small surface element dS in the interior of the contact spot, according to which the differentials of the resultant vector $d\mathbf{F}$ and the moment of friction dM_C with respect to the contact spot center are determined by the formulae [1, 2]: $d\mathbf{F} = -f\sigma\frac{\mathbf{V}}{|\mathbf{V}|}dS,\ dM_C = -f\sigma\frac{\mathbf{r}\times\mathbf{V}}{|\mathbf{V}|}dS,\ \mathbf{V} = (v-\omega y, \omega x), \mathbf{r} = (x, y)$, where f is the coefficient of friction, $\mathbf{r} = (x, y)$ is the position vector of an elemental area in the interior of the contact spot with respect to its center, but ω is the angular velocity of rotation of the contact spot center. Integration of the corresponded differentials over the contact spot yields the resultant vector $\mathbf{F}$ of the friction force and torque $\mathbf{M_C}$.

In addition, in process of the exact integral models construction there are is used well known results from the theory of elasticity that tangent stresses lead to shift in the symmetric diagram of the normal contact stresses in the direction of the instantaneous sliding velocity v or in the rolling direction. To use these results in the dynamics problems, it is proposed the simple asymptotic representations for the contact stresses distributions based on their general properties [3, 4]: $\sigma(x, y) = \sigma_0\left(1 + k_x x/R + k_y y/R\right)$ where R - radius of contact spot and where $\sigma_0 = \sigma_0(r)$ - distribution of normal contact stresses at absence of motion having the properties of central symmetry. To define the corresponded coefficients can be used procedure developed in [5].

If it is assumed that distribution of the normal contact stresses plays a role of density, then the distortion in the central symmetry leads to a shift of the center of gravity of the contact area relative to the geometric center on an amount s in direction of simultaneously sliding because, usually, in the case of the investigation of the aviation pneumatics the directions of rolling and sliding are coincided. This displacement results to the appearance of the rolling moment $\mathbf{M_r}$ parallel directed to the sliding plane, components of which can be calculated by formulas from works [1, 2].

The exact integral model gives a good description of the dry friction effects in the case of combined kinematics, but is inconvenient to be used in problems of dynamics, because it is required to calculate multiple integrals in the right-hand sides of the equations of motion. To escape this difficult procedure, the exact integral model is approximated by smooth analytical functions and has form [2, 4]

$$M_C = \frac{M_0 u}{\sqrt{u^2 + mv^2}},\ F_\parallel = \frac{F_0 v}{\sqrt{v^2 + au^2}},\ F_\perp = \frac{\mu F_0 uv}{\sqrt{(u^2 + bv^2)(v^2 + au^2)}}$$

This model is completely satisfies to the all integral model analytical properties and permits to escape using not smooth functions in the cases when velocities are changed their signs.

Interpolation of the Numerical solution

The quasi-static contact pressure distribution for the real tire of diagonal structure was obtained using finite element simulation for several load cases. As the input data we have the distribution of the contact pressure $p(x)$ over the contact spot Ω computed on the groundwork of the finite element simulation of the contact interaction of the pneumatic tire and the rigid plane. For the two-dimensional finite element model this function depends on the coordinate x and can be tabulated as follows: $p_*^i \equiv p(x_i),\quad x_i \in \Omega,\quad i = 1\ldots n;\quad p(x) = p(-x)$.

We consider the Legendre polynomials as the base system: $\mathrm{p}_k(\xi)$: $\int_{-1}^{1}\mathrm{p}_k(\xi)\mathrm{p}_m(\xi)d\xi = \mathrm{G}_{km} \equiv \delta_{km}\frac{2}{2k+1}$. Taking into account the contact pressure can be written in the following form: $\tilde{p}(x) = p^k\mathrm{p}_k(\xi), [-1,1] \ni \xi = x \big/ \max_{x\in\Omega}(x), k = 0,2\ldots N, N \in \mathbb{N}$.

For the coordinates p^k of the contact pressure approximation $\tilde{p}(x)$ we can pose the following quadratic programming problem: $\min_{\mathrm{p}_k\in\mathbb{R}} \mathrm{G}_{km}\mathrm{p}^k\mathrm{p}^m$ with the first restriction expressed by the inequality $P_k^i p^k \le p_*^i$, $P_k^i = \mathrm{p}_k(x_i)$, $i = 1\ldots n$, $k = 0,2\ldots N$; here P_k^i is the $n\times N$ matrix formed by the columns corresponding to the Legendre polynomials of $0\ldots N-1$ orders and the rows corresponding to the coordinates x_i. The inequality corresponds to the lower approximation of the finite element solution. The second restriction is expressed by the equation $P_k^1 p^k = 0$ that corresponds to the boundary condition $p(x_{\min}) \equiv p^k\mathrm{p}_k(-1) = 0$ at the contact spot boundary. The third restriction is expressed by the equation $p^k\int_{-1}^{1}\mathrm{p}_k(\xi)d\xi = \sum_{i=2}^{n}\frac{p_*^i - p_*^{i-1}}{2}(x_i - x_{i-1})$, that allow one to obtain the normal force and the static friction force computed from the polynomial approximation equal to the one derived from the finite element simulation. The quadratic programming problem was solved numerically by the active set algorithm using the MATLAB code.
Three samples of numerical simulation were used to construct the approximations of the contact pressure. The first one corresponds to the maximum radial deformation of the tire equal to 0.040 m and the contact area diameter equal to 0.087 m, the second one – to the maximum radial deformation equal to 0.029 m and the contact area diameter is equal to 0.080 m, and the third one to the maximum radial deformation equal to 0.020 m and the contact area diameter is equal to 0.068 m. The boost pressure in the tire is equal to $p_* = 190$ kPa.

The dimensionless contact pressure distributions $p(x)/p_*$ were obtained from the finite element solution (100 points) and compared with the computed interpolations for the radial deformation 0.040m, 0.029m and 0.020m.
The obtained solution show that the coefficient a being the inverse of the contact pressure resultant varies diminishes approximately 1,5 times from slightly deformed tire to its full deformed state, whereas the coefficient b diminishes approximately 7,5 times. Therefore the spin moment and the dynamical coupling between the slip and spin of a wheel seems not to be negligible in the analysis of the stability of wheels with combined rolling and sliding.

Conclusions

A new dry friction model is constructed on the basis of the differential formulation of Amonton-Coloumb dry friction law, the improved approximations of integral relationships, and the numerical solution for the contact pressure distributions. This kind of models allows one:

- to take into account the dynamical coupling of sliding and rolling friction for the general kinematics including simultaneous slip, spin, and rolling of a body over the rough rigid surface;
- to replace the very complex exact formulas for the resultant force vector and resultant moment of dry friction obtained by the integration over contact spots by the simple ratios of linear forms to square roots of the quadratic forms with no significant loss of accuracy;

to take into account contact distributions for real objects like pneumatics obtained from specific tests or numerical simulations using the polynomial interpolations that can be easily introduced into the proposed approximated formulae for the resultant force vector and moment of dry friction.

Acknowledgements

This research was financially supported by the Russian Foundation for Basic Researches under grant No. 16-01-00809-a.

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