A Modified Two-Timescale Incremental Harmonic Balance Method for Steady-State Quasi-Periodic Responses of Nonlinear Systems

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Summary. A modified two-timescale incremental harmonic balance (IHB) method is introduced to obtain quasi-periodic responses of nonlinear dynamic systems with combinations of two incommensurate base frequencies. Truncated Fourier coefficients of residual vectors of nonlinear algebraic equations are obtained by a frequency mapping-fast Fourier transform procedure and complex two-dimensional integration is avoided. Jacobian matrices are approximated by Broyden's method and resulting nonlinear algebraic equations are solved. These two modifications lead to a significant reduction of calculation time. Two examples: Duffing equation and van der Pol equation, are simulated. Results from the modified two-timescale IHB method are in excellent agreement with those from Runge-Kutta method. The total calculation time of the modified two-timescale IHB method can be more than two orders of magnitude less than that of the original quasi-periodic IHB method when complex nonlinearities exist and high-order harmonic terms are considered.

Abstract

A modified two-timescale incremental harmonic balance (IHB) method is introduced to obtain quasi-periodic responses of nonlinear dynamic systems with combinations of two incommensurate base frequencies. Since the IHB method can accurately simulate nonlinear systems with multi-dimensions and complex and strong nonlinearities, it is widely used for nonlinear analysis. The IHB method is a combination of harmonic balance carried out by Galerkin procedure, and Newton's method. When the original IHB method is used to obtain quasi-periodic responses of nonlinear systems, a two-dimensional (2D) Galerkin procedure is used to balance truncated multi-dimensional Fourier series to obtain Jacobian matrix and the residual vector in Newton's method. Since multiple integrals need to be calculated in the original IHB method, computation complexity can be extremely high in solving quasi-periodic responses of systems with multi-dimensions and complex nonlinearities, which limits its applications in the quasi-periodic analysis. While using the discrete Fourier transform (DFT) to approximate coefficients of Fourier series can significantly improve efficiency of the IHB method for period responses, directly using the 2D DFT in the IHB method for quasi-period responses is still time- and memory-consuming.

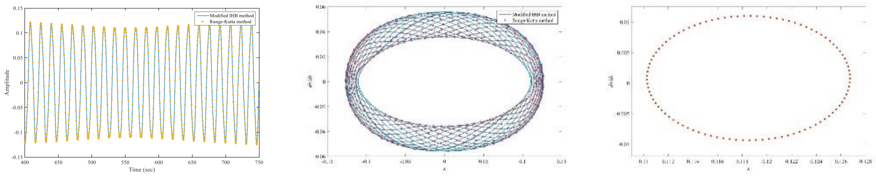
A frequency mapping method (FMM) is introduced in the work, which is a frequency-domain single-valued mapping strategy that can map the truncated 2D Fourier spectrum into a one-dimensional (1D) display spectrum. A partial mapping strategy is then used instead of converting the full 2D spectrum into a 1D one. Through this way, the length of sampling vectors can be decreased to four-ninths of it. The one-dimensional DFT can be used to obtain truncated Fourier coefficients, and complex two-dimensional integration is avoided. The above procedure leads to a significant acceleration in the harmonic balance procedure. In addition, Broyden's method is used to replace Newton's method and Jacobian matrices do not need to accurately calculate in each iteration, and computation time can be greatly reduced.

Two examples: Duffing equation (Eq. (1)) and van der Pol equation with quadratic and cubic nonlinear terms (Eq. (2)), both with two external excitations, are simulated.

$$\ddot{x}+0.04\dot{x}+x+x^3=0.1\cos 0.4t+0.004\cos\sqrt{1.5}t \tag{1}$$

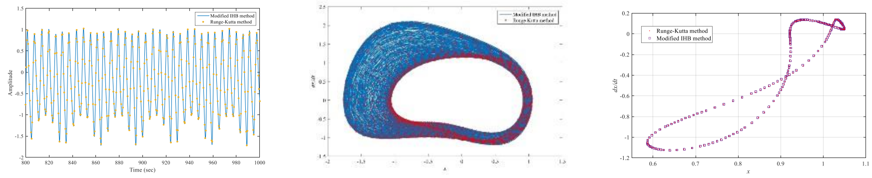
$$\ddot{x}+0.77\left(1-x^2\right)\dot{x}+x+x^2+x^3=0.3\cos 0.2t+0.8\cos\sqrt{1.2}t \tag{2}$$

Simulations are run on a computer with Intel Xeon E3-1230 V3 CPU and 8GB DDR3 1600 RAM using MATLAB R2015a. Results from the modified two-timescale IHB method are in excellent agreement with those from Runge-Kutta method, as shown in Figs. 1 and 2. The total calculation time of the modified two-timescale IHB method and the original IHB method is shown in Table 2. While only odd-numbered 2D harmonic terms are included in Duffing equation using the original IHB method, the modified two-timescale IHB method is 17.25 times faster than the original IHB method. The modified two-timescale IHB method is about 130 times faster than the original IHB method when high-order 2D harmonic terms are included in solving van der Pol equation. Superiority of the modified two-timescale IHB method increases when complex nonlinearities exist and more 2D harmonic terms are included in calculation. The modified two time-scale IHB method has a significant advantage in efficiency over the original IHB method while retaining the same accuracy, and it is suitable for obtaining quasi-periodic responses of systems with multi-dimensions and complex strong nonlinearities.



(a) (b) (c)

Fig. 1 Quasi-periodic response of Duffing equation: (a) time history, (b) phase diagram, and (c) Poincare diagram.



(a) (b) (c)

Fig.2 Quasi-periodic response of van der Pol equation: (a) time history, and (b) phase diagram, and (c) Poincare diagram.

Table 1 Efficiency comparison between the modified two-timescale IHB method and original IHB method by solving Duffing equation

Performance Comparison	Modified IHB Method	Original IHB Method
Truncated Harmonic Terms	31	17
Iteration Numbers	9	4
Calculation Time (ms)	11.8	203.6

Table 2 Efficiency comparison between the modified two-timescale IHB method and original IHB method by solving van der Pol equation

	Modified IHB Method				Original IHB Method			
Truncated Orders	$p = 3$	$p = 5$	$p = 7$	$p = 9$	$p = 3$	$p = 5$	$p = 7$	$p = 9$
Iteration Numbers	11	11	12	12	8	8	8	8
Calculation Time (ms)	4.0	13.9	52.64	107.9	336.6	1979	5839	14015