Perturbation Analysis on the Dynamic Behaviours of Planetary Gear Sets with Friction

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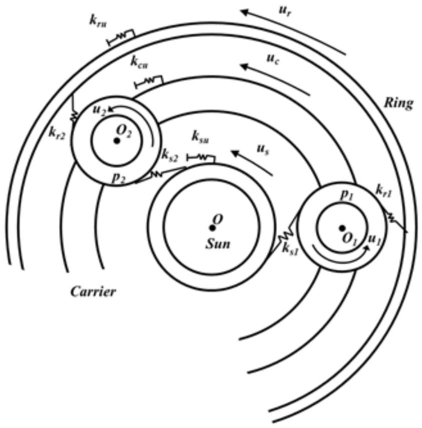
Summary. Sliding friction at the tooth contact is believed one of the main excitations for vibrations and noise for gear transmission systems. The direction of sliding resistance of friction between the mesh teeth is not the same as the primary motion direction or degree of freedom. That results the problem of friction at the tooth contact is different from the classical friction oscillators. In this study, the function denoting the direction of friction is expressed as Fourier series with the consideration of separation of mesh pair. And the dynamic behaviours of planetary gear sets with the effects of sliding friction forces are analysed by the method of multiple scales. Numerical simulations are carried out to confirm the analytical results.

Introduction

Planetary gears have numerous applications in a variety of industrial machinery because of the advantages such as high transmission ratio, high power density, compactness, low bearing load and so on. In gear transmission systems, sliding friction at the tooth contact not only leads power loss [1-], but also is recognized as one of excitations for vibration and noise. Sliding friction changes the time-varying mesh stiffness [2], causes time-varying bearing forces [3], induces nonlinearities [4-5], affects system stabilities [6], and enhances the dynamic transmission error amplitudes [7]. Friction forces are acting in two different directions for approach process and recess process during a single mesh period. It makes the analytical method ineffective. In the current study, the function of directions of friction forces are expressed in Fourier series and the method of multiple scales is used to analyze the effects of mesh friction forces on the dynamic behaviors of planetary gear systems. Time-varying mesh stiffness and backlash nonlinearity are also considered. Numerical integration is employed to verify the proposed method.

MODELING AND ANALYTICAL FORMULAITONS OF PERIOD-ONE MOTIONS

A schematic diagram of an *N*-planet system with the purely rotational vibration is shown in Fig. 1. As shown in Fig.2, the directions of friction forces change at the pitch point P. During the whole mesh period, the arm of the friction force (h) is time-varying as the contact point move along the line of action.



arFig. 1: Lumped parameter model of planetary gear trains

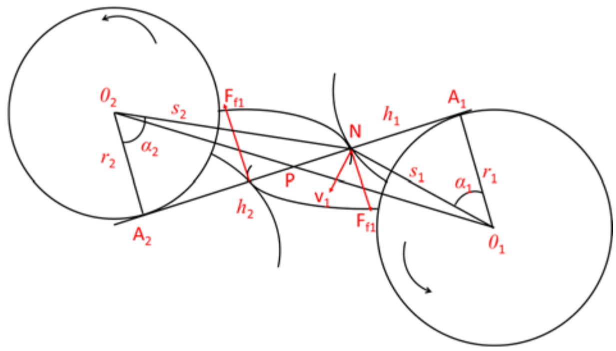


Fig. 2: Time-varying friction directions

The equation of motions is

$$\mathbf{M}\ddot{\mathbf{x}} + [\mathbf{K}_b + \mathbf{K}_m(\mathbf{x}, t)]\mathbf{x} + \mathbf{F}_f(t) - \mathbf{F}_m(t) = \mathbf{F}_t \quad (1)$$

$$\mathbf{F}_f = \mathbf{F}_{fsp} + \mathbf{F}_{fps} + \mathbf{F}_{frp} + \mathbf{F}_{fpr} = \mu_0 \sum_{p=1}^{N} \left[\Gamma_s \Theta_s k_{sp}(t)\big(h_{sp}(t)\mathbf{S}_s + h_{ps}(t)\mathbf{S}_p\big)\mathbf{K}_{sp} + s \to r\right]\mathbf{x} \quad (2)$$

Where $\mathbf{K}_b$ is the supporting stiffness matrix. $\mathbf{K}_m$ is the time-varying mesh stiffness matrix. $\mathbf{F}_f$ is the friction forces vector, $\mathbf{F}_m$ is the additional mesh forces induced by tooth profile modifications. And $\mathbf{F}_t$ is the force vector of applied torque. As expressed in Eq.(2), the friction forces is the functions of mesh stiffness k_{sp}, force arms $h(t)$, tooth separation functions Θ, and friction force directions Γ.

All the time-varying variables aforementioned are expressed as Fourier coefficients, and the method of multiple scales is used to obtain analytical frequency response functions for primary resonances, as expressed in Eq.(3). The friction terms appear in all of the expressions of Ξ_j(j=1-4), which means the friction force has obvious effects on the vibration amplitudes. By vanishing the square root in Eq.(3), one can obtain the peak resonant amplitude for the ith mode as $a_i^{peak} = \frac{\sqrt{\Xi_3^2+\Xi_4^2}}{\omega_i^2\zeta_i}$, which is also a function of friction forces.

$$\omega = \omega_i + \frac{1}{2\omega_i a_i}\left[\Xi_1 a_i + 2\Xi_2 \pm 2\sqrt{\Xi_3^2 + \Xi_4^2 - \left(\omega_i^2 \zeta_i a_i\right)^2}\right] \tag{3}$$

$$\begin{aligned}\Xi_3 = \Bigg\{ & \sum_{w=1}^{L}\sum_{p=1}^{N}\left[k_{sp}^{(1)} G_{spiw} + s \to r\right]\frac{f_{tw}+f_{lw}}{\omega_w^2} - \sum_{p=1}^{N}\left\{\left(\bar{l}_{sp} k_{sp}^{(1)} + \bar{k}_{sp}\, l_{sp}^{(1)} \cos\varsigma_s\right) T_{spi} + s \to r\right\} \\ & + \mu_0 \sum_{w=1}^{L}\sum_{p=1}^{N}\left[\left(\bar{h}_{sp} k_{sp}^{(1)} \cos\varphi_s + \bar{h}_{sp} k_{sp}^{(1)} + \bar{k}_{sp} h_{sp}^{(1)} \cos\tau_{sp}\right) Y_{siw} + s \to r\right]\frac{f_{tw}+f_{lw}}{\omega_w^2} \\ & + \mu_0 \sum_{w=1}^{L}\sum_{p=1}^{N}\left[\left(\bar{h}_{ps} k_{sp}^{(1)} \cos\varphi_s + \bar{h}_{ps} k_{sp}^{(1)} + \bar{k}_{sp} h_{ps}^{(1)} \cos\tau_{ps}\right) Y_{spiw} + s \to r\right]\frac{f_{tw}+f_{lw}}{\omega_w^2}\Bigg\}\end{aligned}$$

$$\begin{aligned}\Xi_4 = \Bigg\{ & \sum_{p=1}^{N}\left\{\bar{k}_{sp} l_{sp}^{(1)} \sin\varsigma_s\, T_{spi} + s \to r\right\} + \mu_0 \sum_{w=1}^{L}\sum_{p=1}^{N}\left[\left(\bar{h}_{sp} k_{sp}^{(1)} \gamma_s^{(1)} \sin\varphi_s - \bar{k}_{sp} h_{sp}^{(1)} \sin\tau_{sp}\right) Y_{siw} + s \to r\right]\frac{f_{tw}+f_{lw}}{\omega_w^2} \\ & + \mu_0 \sum_{w=1}^{L}\sum_{p=1}^{N}\left[\left(\bar{h}_{ps} k_{sp}^{(1)} \gamma_s^{(1)} \sin\varphi_s - \bar{k}_{sp} h_{ps}^{(1)} \sin\tau_{ps}\right) Y_{spiw} + s \to r\right]\frac{f_{tw}+f_{lw}}{\omega_w^2}\Bigg\}\end{aligned}$$

RESULTS AND DISCUSSIONS

An example planetary gear sets with 5 planets is used to investigate the effects of friction forces. Results obtained through multiple scale method into the effects of frictions and dynamic transmission error of various mesh frequency are presented. Numerical integration is used to verify the results obtained by the method of multiple scales and explore the nonlinear phenomena. The planet gears are equally spaced and all of the five sun-planet meshes and the ring-planet meshes are in-phase.

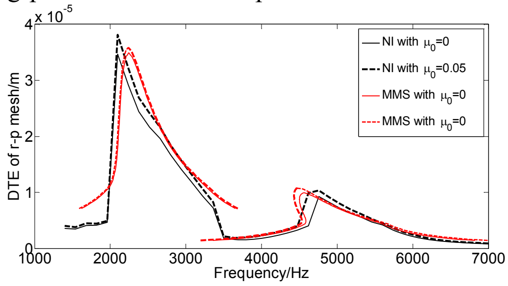


Fig. 3: *DTE of r-p mesh without tooth profile modification*

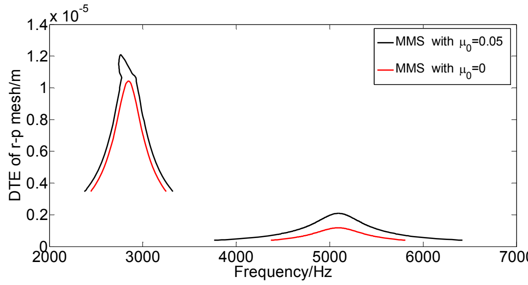


Fig. 4: *DTE of r-p mesh with tooth profile modification*

As shown in Fig.3, with the friction coefficient $\mu_0 = 0.05$, the DTE is enlarged at whole frequency ranges, especially near the two rotational natural frequencies. The friction forces also increase the frequency ranges of tooth separations, which means the non-linearity is strengthened. With the tooth profile modifications applied, the tooth friction forces also enlarge the amplitudes of DTE, as shown in Fig.4. In some conditions, the friction may cause additional tooth separation. That means more tip relives are required to compensate the excitation effects of friction forces. The more details will be presented at the conference.

CONCUDING REMARKS

In this paper, all the time-varying variables including mesh stiffness, tooth separation functions, friction force directions and friction force arms are expanded as Fourier coefficients forms. Then the method of multiple scales is used to investigate the effects of friction on the dynamic behaviors of planetary gear sets. The results show that the friction forces are one of the excitations of vibrations, and more tip relives are needed due to the effects of friction forces.

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