Simplified model of rocking suitcases

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Summary. A simplified mechanical model is presented for the analysis of the rocking motion of wheeled suitcases. The governing equations of the non-smooth non-holonomic system are derived using appropriate approximations with respect to the wheel geometries. Numerical simulations are used to investigate the domain of attraction of the rectilinear motion. The effects of the geometrical parameters are investigated and critical setups and towing speed domains are detected.

Introduction

The rocking motion of towed wheeled suitcases is one of the most irritating phenomenon that can disturb travellers. On the contrary, this phenomenon is very exciting in mechanical sense since the mechanical and mathematical modelling of the problem is complex. On the one hand, the suitcase makes spatial motion that is governed by the kinematic constraints of the rolling wheels. Namely, a non-holonomic system is in question, and the derivation of the equations of motion needs special attention. On the other hand, after the governing equations are determined one obtains a non-smooth system due to the fact that different equations are valid for the rolling on the left and right wheels of the suitcase. All this makes the analyses of the stability of the rectilinear motion non-classical.
In the literature, some reports can be found on this topic. Mechanical models are constructed and theoretical analyses are performed in [1, 2]. In [3], the effect of the human control is also considered in a simplified in-plane model. Here, we deal a simplified spatial model, by which the effects of the geometrical parameters of the suitcase are investigated numerically.

Mechanical model

The simplified mechanical model is shown in Fig. 1. We assumed that the body of the suitcase is rigid and we model it by means of a lumped mass m at its center of mass C. Parameters e and f are used to describe the position of the center of mass, while l and b refers to the length and to the half width of the suitcase. The suitcase is pulled by a ball-joint at A with constant speed v. The parameter h characterizes the distance of the towing point A and the ground.

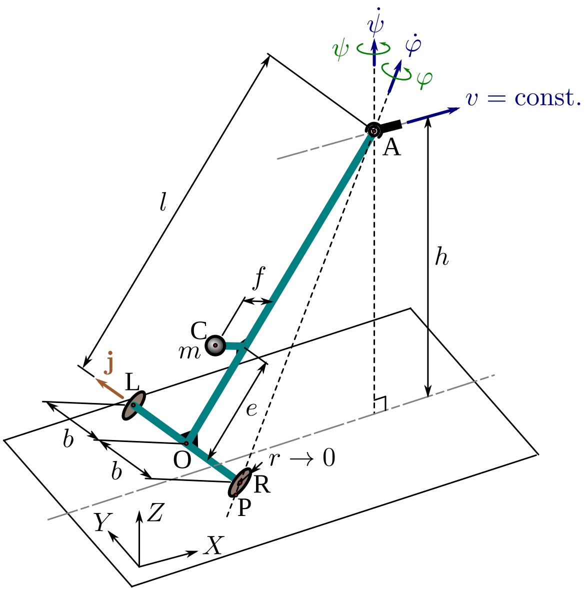

Figure 1: The simplified mechanical model of the towed suitcase

Four different states of the suitcase can be differentiated during the motion: both wheels contact to the ground, the right or the left wheel contacts only, non of the wheels have contact with the ground. In the first case, the suitcase makes in-plane motion and one second-order ordinary differential equation describes the motion of the suitcase if rolling is considered. One can prove that the rectilinear motion is linearly stable for any speed if the suitcase is towed, and it is unstable if the suitcase is pushed. This statement is only valid if the ball-joint at A is rigid. See [4], where the effect of the elasticity of the towing suspension is investigated.
In point of view of derivation of the equation of motion, the most complicated scenarios are related to the cases when one of the wheels has contact only. Namely, the suitcase makes spatial motion meanwhile the kinematic constraint of rolling is also present. In order to obtain the equations of motion in more manageable form for this case, we assume that the

radii of the wheels are small. Hence, the points A and R determines an axis around it the suitcase can rotate when the left wheel lifts up. Namely, the tilting angle $\varphi(t)$ can be used as generalized coordinate together with the yaw angle $\psi(t)$ that describe the rotations around the vertical Z axis (see Fig. 1).
The kinematic constraint of rolling means that the contact point of the wheel attached to the ground has zero velocity, i.e. $\mathbf{v}_{\mathrm{P}} = \mathbf{0}$. Since zero radius of the wheel is considered (i.e. $\mathrm{R} \to \mathrm{P}$), the kinematic constraint of rolling can be rephrased as

$$\mathbf{v}_{\mathrm{R}} \cdot \mathbf{j} = 0 \,, \tag{1}$$

where the unit vector $\mathbf{j}$ moves together with the suitcase and directs always to the lateral direction.
The equations of motion are derived with the help of the Lagrange-equation of the second kind extended by a Lagrange-multiplier for the constraining force of the rolling contact. Here, we do not present the derivation process and the equations due to the space limitation. But, one can obtain a system of three first order ordinary differential equations (with the state variables $\varphi(t)$, $\omega(t) := \dot{\varphi}(t)$ and $\psi(t)$) that describe the motion of the suitcase when one wheel has contact only. The governing equation can be determined either for the left wheel in contact and for the right wheel in contact cases. If the towed suitcase is rocking, switching between these governing equation is required when the non-contacted wheel has an impact with the ground. The energy dissipation of this impact is taken into account in our model by means of the coefficient of restitution C_{R} by which:

$$\omega^{+} = C_{\mathrm{R}} \omega^{-} \,, \tag{2}$$

where ω^{-} and ω^{+} are the angular speeds around the different (left and right) tilting axes before and after the impact, respectively.

Numerical investigation

Governing equations and the impact model were implemented in computer code and simulations were run for different parameters and initial conditions ($\varphi(0) = \varphi_0$, $\omega(0) = 0$, $\psi(0) = 0$). Figure 2 shows the domain of attraction of the rectilinear motion for realistic system parameters. Green circles refers to the cases when the rocking motion tends to the zero amplitude while red dots represents growing vibration amplitudes. As it can be seen, there is a narrow speed range where vibrations decay even for large initial tilting angles.

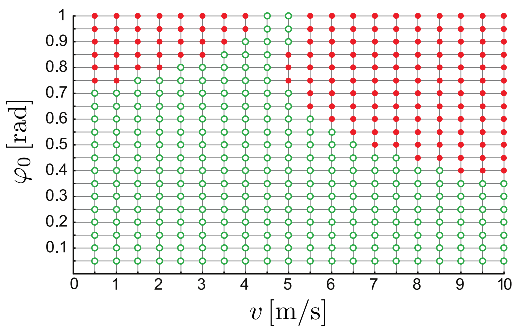

Figure 2: The domain of attraction of the rectilinear motion.

Conclusions

A simplified mechanical model is constructed by which the rocking motion of the towed wheeled suitcases can be efficiently analyzed. The effect of the geometrical parameters are investigated and critical parameter setups are detected. Theoretical results are also compared to laboratory experiments to validate the model.

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