Semi-analytical investigation of unsteady free-boundary flows

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Summary. The methods which are intermediate between purely analytical and purely computational ones belong to a semi-analytical group, according to the classification by Milton Van Dyke. They are mainly the methods of processing the series and sequences. This group comprises the well-known algorithms based on the Padé approximants and continuous fractions as well as convergence acceleration algorithms, Domb-Sykes test, etc. The authors use this technique to investigate unsteady inviscid incompressible free-boundary flows. The solution is obtained in the form of a power series in time, the coefficients for it being computed. The obtained series are summarized and singularities are revealed.

Free-boundary flows

A general problem which may be solved using power series in time consists in finding deformations of the domain occupied by a liquid if pressure of the free boundary is constant (surface tension is absent). An initial configuration of the domain and an initial velocity field are assumed to be known.

Conformal mapping

Conformal mapping of the fixed domain onto the flow region is sought in the form of power series or the Laurent series. About a thousand of terms of a series with length of mantissa 1300 decimals are found. Series are summed by using the technique proposed in [1]. The change of variables of special form with subsequent Padé summation is used. The free boundary, velocity characteristics and other parameters of the flow at each moment of time are found.

Inertial motion of a cylindrical cavity

The greatest attention devoted to the problems of deformation of cavities in infinite liquid. It is known that in such flows cumulative jets often arise [2]. The simple flow in which such cumulative jet is realized is an inertial motion of cylindrical cavity in a liquid.

Let for time $t = 0$ there be a circular cylindrical cavity of unit radius in a liquid which moves with a unit velocity (liquid at infinity is at rest). It is necessary to find a necessive deformation of the cavity when $t > 0$ if zero pressure is maintained inside it and the cavity volume is constant. The problem was first considered in [3], where the free surface form is defined on the basic of finite-difference methods.

The data on singularities in the plane of complex time, which restrict the convergence of power series were mainly obtained, using Padé diagram. The zeros and poles of the Padé approximant were noticed to arrange along some lines, cuts, the tops of which show to a singularity. There are 5 singular points. One cut corresponds to a movable singular point. Its site is different for different points in a liquid. The remaining four cuts are unmovable. The behavior of the singularities are demonstrated in Fig. 1. The free surface form at different time moments obtained by the Pade-summation after change of variables is shown in Fig. 2. The pressure at a greate distance from the cavity is shown in Fig. 3.

Conclusions

The proposed model seems to be very promising. Although complicated in terms of application, it admits a deep research of the problem. It allows a free surface to be constructed for large times and the appearance of singularities on it to be recorded.

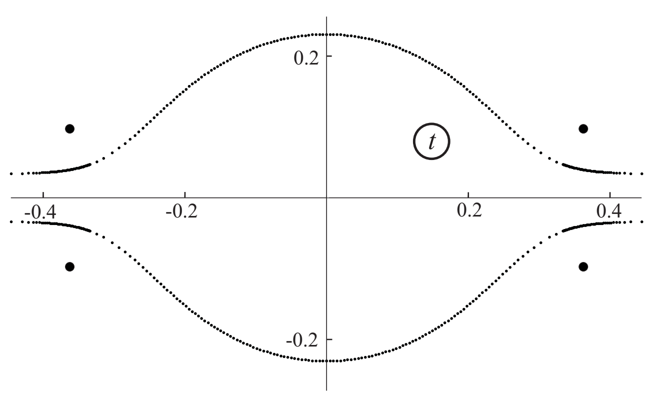

Figure 1: The site of singularities in the complex plane t. The circlets denote unmovable singularities. The points are correspond to a movable singularity for the free surface.

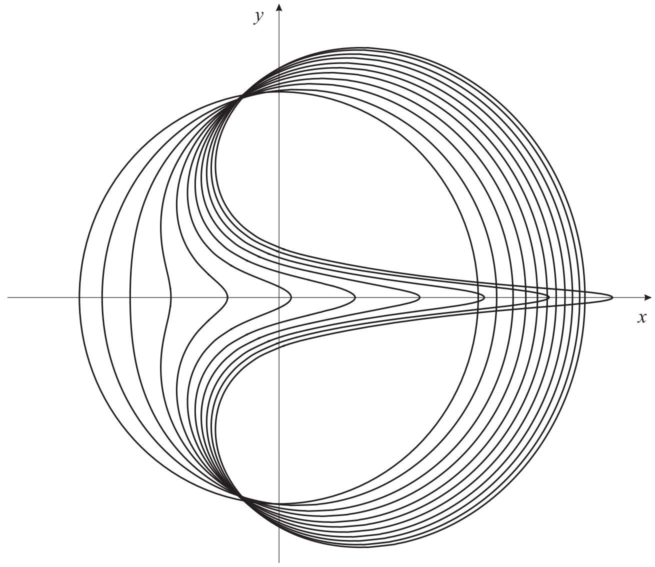

Figure 2: Time evolution of the free surface.

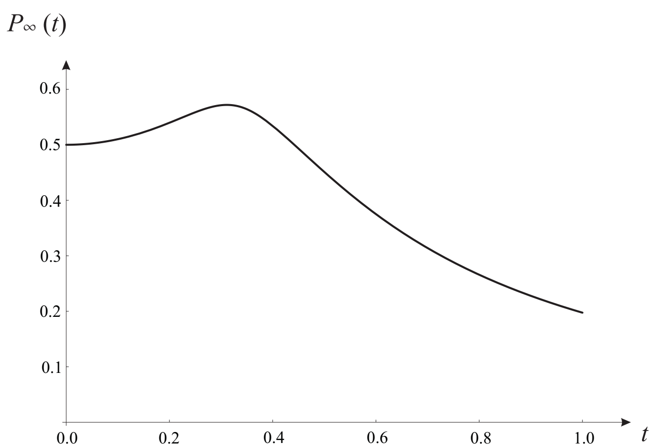

Figure 3: Pressure dependence on time at infinity: $\sqrt{x^2+y^2} \to \infty$.

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