Classic problems of linear acoustics

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THE ERROR OF THE ACOUSTIC METHOD for AIR FLOW MEASUREMENT IN MINES

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The problem of the propagation time of a sound signal between two given points A and B in stationary gas flow is considered. It is shown that the gas flow rate modifies the time of the signal reception by an amount proportional to the flow rate regardless of the detailed speed profile. The time difference in receiving signals from point to point upstream and downstream with high accuracy proportional to the air flow rate. It is shown that the relativeerror of the formula does not exceed the maximum square of the gas stream Mach number.
This allows to measure the gas flow rate in the working with an arbitrary stationary subsonic velocity field.

Keywords: acoustic beam, Fermat's principle, the gas flow rate, steady subsonic velocity field.

INTRODUCTION

The establishment of dispatching and automatic control systems for mines ventilation is impossible without the availability of perfect air flow sensors. Existing anemometers (tachometer, heat) do not meet these requirements. The measurement error of the average in the cross section velocity by such sensors is about 15-20, sometimes reaching 30%. The reason for this is that the velocity measured in one point is interpreted as the average over the cross section.

The reliability of the sensors is small, because they are exposed to the damaging effect of very aggressive atmosphere. Mounted on the roof so called “point” clutter cross section generation anemometers, often are not possible to use. Many commercial proposals for the acquisition of flowmeters suitable for the flow measurement in particular for wide ducts, are possible to find in Yandex'e typing "time-of-flight flow meters".

One of the authors of this article S. Z. Shkundin proposed a investigated different schemes (current, time-pulse, phase methods) for measurement of the average flow rate parameters. Their principle of operation is based on the fact that the propagation time of the acoustic oscillations through the air stream depends on the velocity of the moving air from the radiator to the receiver. Recording the changes in that time, one can determine the speed of a moving stream. Acoustic transducers are located on opposite walls of the working not diverting its cross section [1,2,3]. Due to the fact that the acoustic beam crosses the entire plot of the velocity distribution in the flow, the acoustic anemometer measures the average over its trajectory speed of the air flow with an error not exceeding 5% (in pre-experimental estimates).

The evaluation of such flow rate measurements relative error for arbitrary velocity curve is given in this article.

It should be noted that the study of the air flow velocity influence upon the phase velocity of the acoustic wave is a complex mathematical problem. Some special cases of this problem solution were studied earlier. The phase velocity of plane waves in a circular pipe filled with a moving medium with power law of velocity change along the radius of the tube has been numerically investigated. The solution of the wave equation carried out by the method of sampling in which the entire volume of the pipe was divided into separate cylinders, in each of which the flow velocity was considered constant, which allowed to reduce the wave task to the solution of the Helmholtz equations in each cylinder[4] . The numerical results of the calculation of phase velocities of flat quasi-homogeneous and inhomogeneous waves in the pipe for various velocities of the moving media have been obtained and analyzed. It is shown that the variation of the phase velocity of a homogeneous plane wave in the pipe associated with the movement of the medium is equal to the average velocity ofthe flow for various air dynamics curves in the tube.

The increment of the phase velocity of a plane wave in a pipe with a moving acoustic medium relative to the phase velocity of a homogeneous flat wave in a pipe with a stationary medium is approximately equal to the average flow velocity in the pipe with the difference of about a few percent. In [4] analytically studied the acoustic waves in a circular tube generated by a ring vibrator, placed in a uniform subsonic flow codirectional with the axis of the waveguide. The method of Wiener-Hopf solution in the form of expansion in series with Bessel functions have been used.

METHOD for MEASUREMENT OF GAS FLOW RATES IN MINE WORKING

The time-pulse method for theaverage in cross section flow velocity measurement is described in monograph [1] and consists in the following. The generator supplying the transducers produces an electrical signal. Both converters simultaneously emit acoustic pulses towards each other and immediately switch to reception. The speed of propagation of the pulse downstream is equal to the sum vector of the speed of sound in theairand the velocity vector of the flow. The speed of pulse against the flowis equal to the difference of these vectors. Pulses are applied to transducers operating in the receive modenot simultaneously, and the interval between their receptions (which is the parameter to be measured) is proportional to the flow rate. The impulse coming downstream switches on a time-measuring circuit, and the pulse passing against the flow, turning it off.The dependence of the difference of time intervals of the acoustic signal passage from point A to point B and back upon the speed of the homogeneous air flow may be calculated.
While measuring of air flow in the working vector of the acoustic beam is a summand of a vector of sound speed and the air flow rate vector. In the presence of a homogeneous air flow the audio signal also travels by the shortest straight-line path, but with a speed dependent on the speed of the air flow.

The dependence of the transmission time intervalsdifference of the acoustic signals propagating from the point A to the point B and back on theuniform airflow speed **u** can be derived as follows. Considering a channel of rectangular cross section of width **b** and height **h** of the vertical walls. Longitudinal section of the working is presented on the Fig. 1. A point is located at the beginning of a Cartesian coordinate system, the point B has coordinates a,b.

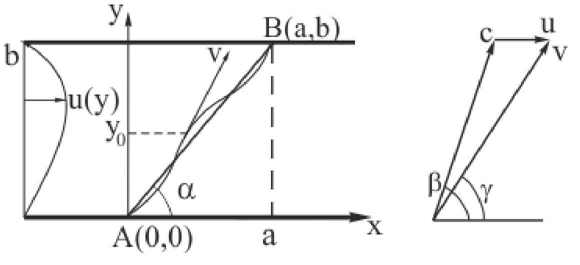

Fig. 1 The scheme ofthe air flow measuring in working (left). The acoustic ray vector v is composed of the sound velocity vector **c** and the airflow velocity **u** (right).

If the air in the channel does not move, then the sound signal with speed **c** passes to the receiver during time $t_0 = \sqrt{a^2 + b^2} / c$, spreading along the shortest straight path AB.

In the presence of a homogeneous airflow $u(y)$ = const, the sound signal also travels the path AB along the shortest straight path, but at a speed v that depends on the speed of the air flow, under the condition that the radiator diagram has a sufficient width. The speed v is determined as follows.

The angle $\beta - \gamma$ between the vectors **c** and **v** by the sine theorem is $\sin(\beta - \gamma) / u = \sin\gamma / c$ so expressingthe angle β through γ (see Fig. 1 on the right). Hence we obtain

$$\sin(\beta - \gamma) = M \sin\gamma, \quad \cos\beta = \cos(\gamma + (\beta - \gamma)) = \cos\gamma(1 - M^2 \sin^2\gamma)^{1/2} - M \sin^2\gamma \qquad (1)$$

The length of the vector **v** is determined by the cosine theorem: $v^2 = c^2 + u^2 + 2cu\cos\beta$

The time ofthe signal passage is equal to $t_{AB} = \sqrt{a^2 + b^2} / v$, whose expansion in the Mach number has the form

$$t_{AB} = t_0 + \frac{1}{c}\left(-aM + \frac{2a^2 + b^2}{2\sqrt{a^2 + b^2}} M^2 - aM^3 \right)$$

To measure the air speed **u,** the difference in the reception times $\Delta t = t_{BA} - t_{AB}$ of signals from the inverse ray path from point B to point A and along a straight trajectory from point A to point B is determined. The times t_{AB} and t_{BA} are distinguished by the sign of the Mach number and the terms of the expansions for odd powers of the Mach number have opposite signs, and for even powers coincide. Therefore, the expansionof Δt includes terms only even powers.

$$\Delta t = \frac{2a}{c}\left(M + M^3 + O(M^4) \right) = \frac{2a}{bc^2} Q(1 + M^2 + O(M^4)), \quad Q = \int_0^b u(y)dy = bu$$

Thus, for a homogeneous flow rate Q is measured through the difference in the arrival times of the pulses to the converters according to the formula

$$Q = \frac{bc^2}{2a} \Delta t \left(1 + \Delta\right) \qquad (2)$$

The relative error is equal to the square of the Mach number $\Delta = -M^2 + O(M^4)$. Formula (2) is used as the basis forair consumption in mines measurement by method [1, 2]. It remains unclear how the inhomogeneity of the velocity $u(y)$ distribution affects the error Δ.

Below we study the difference of time pulses in the air flow with an arbitrary velocity diagram, assuming that the length of the acoustic wave is much smaller than the length of the air duct. For the legitimacy of this assumption, it is necessary to use emitters with a frequency fmore than 10,000 Hz. At this frequency, the length of the acoustic wave $\lambda = c / f$ in the air is less than $340 / 10^4 = 3.4 \times 10^{-2}$ m. The typical widths of air an air duct in the mines is not more than 10 m. Therefore, the condition λ<<l

is satisfied and the laws of geometric optics can be used to propagation of the acoustic beam (see [5], §53).

It is shown that for any velocity diagram $u(y)$ the air flow Q is determined through the difference in arrival times of pulses Δt according to formula (2) with relative error:

$$\Delta = -\bar{M}^2 + 2\frac{a^2}{b^4}\overline{\Delta M^2} - \frac{a^4}{2b^6}\frac{\overline{\Delta M^3}}{\bar{M}} + O(M^4),$$
$$\bar{M} = \frac{1}{b}\int_0^b M(y)dy, \quad \overline{\Delta M^2} = \frac{1}{b}\int_0^b (M(y)-\bar{M})^2 dy, \quad \overline{\Delta M^3} = \frac{1}{b}\int_0^b (M(y)-\bar{M})^3 dy \qquad (3)$$

The relative error is equal to the square of the average Mach number $\bar{M}^2$ and the terms with the quadratic $\overline{\Delta M^2}$ and cubic $\overline{\Delta M^3}$ dispersions of the distribution $M(y)$.For $a = b$ and $a \gg b$ formula (3) is simplified:

$$\Delta = -\bar{M}^2 + 2\overline{\Delta M^2} - \frac{\overline{\Delta M^3}}{2\bar{M}} + O(M^4), \quad a = b$$
$$\Delta = -\frac{a^4}{2b^4}\frac{\overline{\Delta M^3}}{\bar{M}} + O(M^4), \quad a \gg b \qquad (4)$$

The result (4) determines the technical characteristics of the acoustic anemometer for measuring the airflow in the mineworkings according to the procedure proposed in [1, 2].

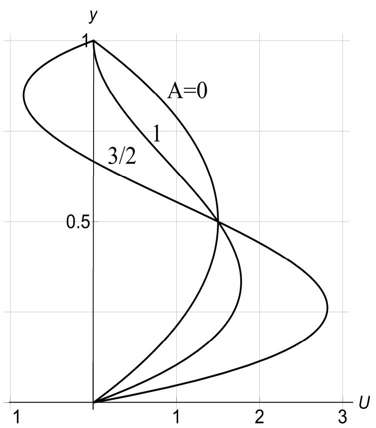

Fig.2. Diagrams of velocities.

Here are some examples of calculating the error of flow rate measurement. To do this, let us consider the following family of speed diagrams:

$$M(y) = u(y)/c = \bar{M}U(y,A), \quad U(y,A) = \frac{1}{B}y(1-y)(1-Ay),$$
$$B = \int_0^1 y(1-y)(1-Ay)dy = \frac{1}{6} - \frac{A}{12}.$$

For this family, the quadratic $\overline{\Delta M^2}$ and cubic $\overline{\Delta M^3}$ dispersionsare the following :

$$\overline{\Delta M^2} = \frac{28-28A+13A^2}{35(2-A)^2}\bar{M}^2, \quad \overline{\Delta M^3} = -\frac{2}{35}\bar{M}^3$$

The graphs of the diagrams when ... $A=0;\ -1$ and $-3/2$ are shown on Fig. 2. For these values, with $a=b$ using (4), we find the following error values:

$$A=0: \quad \Delta \approx -0.57\bar{M}^2, \quad A=1: \quad \Delta \approx -0.23\bar{M}^2, \quad A=3/2: \quad \Delta \approx 2.5\bar{M}^2$$

When $a \ll b$ for all values of A, the error is: $\Delta = \frac{1}{35}\left(\frac{a}{b}\right)^4 \bar{M}^2$.

As can be seen, for all the diagrams, the relative errors for $a=b$ are small in the order of the square of the average Mach number. Exceptions may bethe diagrams with pronounced backflow. For them, the coefficient of $\bar{M}^2$ can grow strongly. The error, however, for $a \gg b$ and non-homogeneous flow contains multiplier $(a/b)^4$ which becomes for $a/b>5$ anomalously large. Therefore, it is not advisable to choose such relations for flow measurement.

We proceed to the derivation of the error formulas (3) and (4). It reduces to a boundary problem for the nonlinear differential equation for the acoustic ray trajectory . Its solution must be constructed in the form of an expansion in Mach number up to the third degree inclusive.

The trajectory of the acoustic beam. The velocity vector v of the signal propagation is a summand of thesound signal vector directed at angle β to axis x and vector $\mathbf{u} = (u(y), 0)$ parallel to axis x (see the right figure 1) $\mathbf{v} = (c\cos\beta + u(y), c\sin\beta)$ with the length

$$v = ((c\cos\beta + u)^2 + (c\sin\beta)^2)^{1/2} = c(1+2M\cos\beta + M^2)^{1/2}, \quad M = u(y)/c \ (5)$$

where M – is the local Mach number, and $\cos\beta$ is expressed in terms of trigonometric functions of the angle γ - inclination of the tangent path of the ray to the axis x.

Let the sound signal be fed at a point $A(0,0)$ located at the origin. The elementary duration of ray movement is defined as $dt = ds/v$, where $ds = dy/\sin\gamma$ – is the elementary segment of the path, directed along the velocity vector v. The time of the signal reception t_{AB} at a point $B(a,b)$ is determined by the summand of all the time intervals dt along the ray trajectory AB.

Using (5), we find
$$t_{AB} = \int_{AB} \frac{ds}{v} = \int_0^b \frac{dy}{c(1+2M\cos\beta + M^2)^{1/2}\sin\gamma} \qquad (6)$$

The trajectory of theacoustic signal propagation is determined from the Fermat minimum principle, which is formulated as follows (see [6], p. 87, [7], p. 374).

Among all paths from point A to point B the actual path of the acoustic signal passes in the shortest time.

The method proposed for solvingthis problem is convenient in connection with the fact that the minimum value of the found functional t_{AB} is the desired quantity.

We are looking for the trajectory of an acoustic ray in the form of a function $x(y)$.

The trigonometric functions of the angle γ are expressed in terms of the derivative of the function $p = dx / dy$.

$$\frac{1}{\sin\gamma} = \sqrt{1+(p)^2}, \quad \frac{\cos\gamma}{\sin\gamma} = p \quad (7)$$

Substituting (7) into the functional (6), let us find its expansion in Mach number with the accuracy up to M^3 inclusive .

$$t_{AB} = \int_0^b \frac{dy}{c} L(p,y), \quad L(p,y) = \sqrt{1+p^2} - M(y)p + \frac{M^2(1+2p^2)}{2\sqrt{1+p^2}} - M^3 p, \quad p = x'(y) \quad (8)$$

The trajectory $x(y)$ is the functional extremis and can be found from the Euler equation for the functional (6) [5]

$$\frac{d}{dy}\frac{\partial L}{\partial p} = 0$$

The beam shape $x(y)$ selected here compares favorably with the traditional $y(x)$ in that equation has the decision

$$\frac{\partial L}{\partial p} = \frac{p}{\sqrt{1+p^2}} - M(y) + \frac{M(y)^2 p(3+2p^2)}{2(1+p^2)^{3/2}} - M(y)^3 = K. \quad (9)$$

where K is some constant.

Despite the fact that the equation can be integrated, it isstill a complex non-linear differential equation with respect to $p = dx / dy$ with an arbitrary function $M(y)$ and indefinite K. The solution $x(y)$ must satisfy the condition $x(b) = a$. Resolution will be found in the form of expansions in terms of Mach number to the cubic terms inclusive.

Let us find the solution of equation (9) with respect to *p*. The solution $p(K,M)$ is found in the form ofMach number expansion up to members of the third degree inclusive.

$$p = \frac{K}{\sqrt{1-K^2}} + \frac{M(y)}{(1-K^2)\sqrt{1-K^2}} + M(y)^2 \frac{4K^3 - K^5}{2(1-K^2)^{5/2}} + M(y)^3 \frac{5K^4}{2(1-K^2)^{7/2}}. \quad (10)$$

Using conditions $x(0) = 0, \quad x(b) = a$ we obtain the equation for constant K :

$$x(b) = \int_0^b p(y)dy = b\left(\frac{K}{\sqrt{1-K^2}} + \frac{\bar{M}}{(1-K^2)^{3/2}} + \overline{M^2}\,\frac{4K^3 - K^5}{2(1-K^2)^{5/2}} + \overline{M^3}\,\frac{5K^4}{2(1-K^2)^{7/2}}\right) = a,$$
$$b\bar{M} = \int_0^b M(y)dy, \quad b\overline{M^2} = \int_0^b (M(y))^2 dy, \quad b\overline{M^3} = \int_0^b (M(y))^3 dy, \qquad (11)$$

where $\bar{M}, \overline{M^2}, \overline{M^3}$ - the average values over the cross section Mach number and its powers.

From equation (11) the constant K can be found as the expansion of the Mach number powers .

$$K = \frac{a}{\sqrt{a^2+b^2}} - \bar{M} + K_2 + K_3, \quad K_2 = \bar{M}^2\,\frac{3a\sqrt{a^2+b^2}}{2b^2} - \overline{M^2}\,\frac{3a^5 + 4a^3b^2}{2b^2(a^2+b^2)^{3/2}},$$
$$K_3 = -\overline{M^3}\,\frac{(5a^4 + 6a^2b^2 + b^4)}{b^4} + \bar{M}\,\overline{M^2}\,\frac{3a^2(5a^2+4b^2)}{2b^4} - \overline{M^3}\,\frac{5a^4}{2b^4} \qquad (12)$$

Pulses time difference. Substituting (12) into (10) and then into the functional (8), we find the required time in the form of an expansion in the number of Mach: zero term t_0 and t_1, t_2, t_3 proportional to the first, second and third degree of Mach number. The difference in time of pulses Δt consists of members with odd powers of the Mach number.

$$t_{AB} = t_0 + t_1 + t_2 + t_3, \quad \Delta t = t_{BA} - t_{AB} = -2(t_1 + t_3), \quad -2t_1 = \frac{2a}{c}\bar{M},$$
$$\Delta = -\frac{t_3}{t_1} = -M^2 + 2\frac{a^2}{b^2}\left(\overline{M^2} - \bar{M}^2\right) - \frac{a^4\left(\overline{M^3} - 3\overline{M^2}\,\bar{M} + 2\bar{M}^3\right)}{2b^4\bar{M}} \qquad (13)$$

The term $-2t_1$ defines the first term in the rate equation (2), the ratio t_3/t_1 is the relative error. For $a = b$ the formula for the relative error is simplified

$$\Delta = -\frac{t_3}{t_1} = -4\bar{M}^2 + \frac{7\overline{M^2}}{2} - \frac{\overline{M^3}}{2\bar{M}} \qquad (14)$$

The expression for the relative error is conveniently expressed in terms of quadratic $\overline{\Delta M^2} = \overline{(M-\bar{M})^2}$ and cubic $\overline{\Delta M^2} = \overline{(M-\bar{M})^2}$ dispersion using the following equalities:

$$\overline{M^2} = \overline{(\bar{M} + \Delta M)^2} = \bar{M}^2 + \overline{\Delta M^2},$$
$$\overline{M^3} = \overline{(\bar{M} + \Delta M)^3} = \bar{M}^3 + 3\bar{M}\,\overline{\Delta M^2} + \overline{\Delta M^3}$$

Substituting these expressions in (13) and (14)we get the desired formulas (3) and (4).

The proof is finished.

CONCLUSION

Regardless of the detailed distribution of the air velocity in the working section the airflow rate is almost proportional to the difference signal receiving pulses Δt from point B to point A, and vice versa. The relative error of time-pulse flow measurement consists of the difference between the relative error of pulses Δt and the relative error Δ of proportionality of the law (2), expressed by the formula (3) and (4). The difference between the pulses Δt, as follows from the formula (2) proportional longitudinal dimension *a*. To improve the accuracy of the measurement of this quantity *a* should be increased. However, for $a \gg b$ the value of error contains, according to (4), an abnormally large multiplier $(a/b)^4$ for cubic dispersion of nonuniform velocity profile. Therefore, the ratio $a/b > 5$ chosen is inappropriate. The $a/b = 1$ relation seems to be optimal for the most accurate flow measurement.

The relative error Δ is of order of Mach number square. It depends not only on the average value of the Mach number, but on the velocity profile form .Formulas (3) and (4)obtained for the relative error can be used as the technical characteristics of the proposed method anemometry.

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