Balancing on accelerating skateboard

Balazs Varszegi*, Denes Takacs** and Tamas Insperger *
*Department of Applied Mechanics, Budapest University of Technology and Economics and
MTA-BME Lendulet Human Balancing Research Group, Budapest, Hungary
** MTA-BME Research Group on Dynamics of Machines and Vehicles, Budapest, Hungary

Summary. The aim of this study is to investigate the dynamics of skater-board system under acceleration, as a model of a downhill motion. The mathematical model is a system of time-varying neutral delay-differential equation. In this study, the so-called frozen time method is used and the method is verified by means of numerical simulations. It is shown that the varying longitudinal speed can explain the loss of stability of skateboards at higher speeds. The investigation of the magnitude of the board's longitudinal acceleration highlights the positive effect of the higher acceleration.

Introduction

Analysis of skateboards' stability problems started in the late 1970s (see [1] and [2]), but still nowadays, there are some unexplored phenomenon in this field, for example the vibrations induced at high speeds. This study aims to investigate the behaviour of skateboards under constant longitudinal acceleration, as a model of a downhill motion. Usually, in vehicle dynamics, varying speed leads to time-varying coefficients in the governing differential equations, and the corresponding stability analysis is therefore a challenging task. In case of skateboarding, the reaction time of the human is modelled by a feedback delay, thus the system under analysis is a delay-differential equations with varying coefficient.

Investigated model

The mechanical model in question (see Fig. 1) is based on [3]. This contains two connected massless rods, one stands for the board with the length of $2l$ and the other one models the skater with height h, which has a lumped mass m at its free end. Due to the connection of the wheels and the ground, the position of the system can be described by five generalized coordinates; $x(t)$ and $y(t)$ denote the horizontal position of the centre (S) of the board, while b just parametrizes the position of the skater on the board; $\psi(t)$ stands for the longitudinal direction of the board; $\varphi(t)$ and $\beta(t)$ are the tilting angle of the skater and the board, respectively. The plane motion of the massless board is limited by three kinematic constraints: regarding to the rolling wheels of the skateboard, the direction of the velocity of points F (front) and R (rear) are given as a function of the tilting angle β and the angle between the pivot axis and the ground κ (see Fig.1.b); and the longitudinal speed of the board is prescribed as a linear function of time $v(t) = at + v_0$. The free rotation of the board is prevented by a torsional spring with stiffness k_t.

The skater-board interaction, as a human balancing mechanism, is modelled by an active (controlled) and a passive (uncontrolled) torques at the ankle. The source of the passive torque is the muscle stiffness (k) originated in the muscle stretching due to the ankle rotation. The active torque is due to delayed neuromuscular sensory feedback from the proprioceptive, vestibular and visual systems. Since, these sensory systems provides information about the absolute body angle (φ) with respect to the vertical direction and the angular velocity ($\dot\varphi$), the corrective torque reads as

$$M_\mathrm{h}(t) := k\left(\varphi(t) - \beta(t)\right) + p\varphi(t-\tau) + d\dot\varphi(t-\tau)\ ,$$

where p and d are the control gains and τ is the reflex delay of the skater.

Since this is a nonholonomic system, the Gibbs-Appell method is used to derive the equation of motion with the pseudo velocity $\sigma(t) := \dot\varphi(t)$ similarly as in [3]. The resulting governing equation is a nonlinear, time-varying (explicitly time-dependent), neutral delay-differential equation.

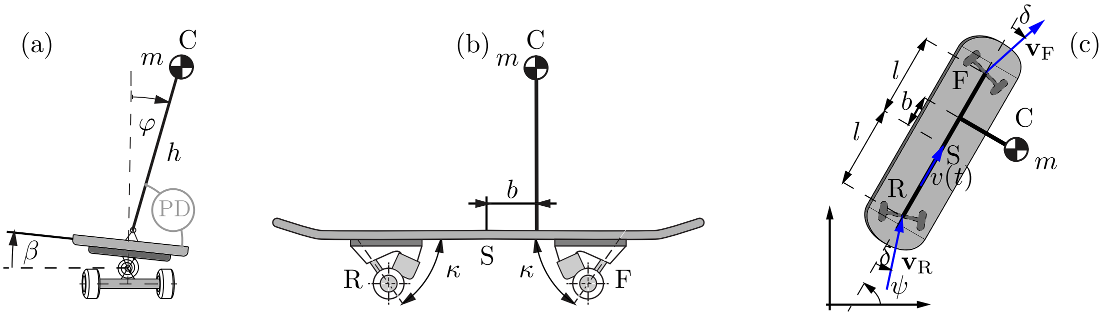

Figure 1: Mechanical model: (a) back view, (b) side view, (c) upper view

Stability analysis

The mathematical model remains a time-varying neutral delay-differential equation after linearisation around the trivial solution (i.e. $\varphi \equiv 0$, $\sigma \equiv 0$) with respect to small perturbations in σ and φ. Stability problems of these systems are not as well studied as that of time invariant systems, since the examination time is finite. Here, the stability analysis is organised as follows. First, the frozen time eigenvalue method is considered. Namely, the variation of the time dependent coefficient is neglected (the time is frozen for these coefficients) and the stability is studied by the analysis of the eigenvalues similarly to time-independent systems. Then, the results are verified by means of numerical simulations.

In case of the frozen time linear stability analysis, the trivial equilibrium for a given pair (P, D) of dimensionless control gains (where $P = \frac{p}{k_t}$ and $D = \frac{d}{k_t}\sqrt{\frac{g}{h}}$ according to [3]) is called stable during the accelerating movement, if the real part of all of the corresponding characteristic exponents is negative for any investigated longitudinal speed. The studied speed domain is 5 km/h $< v(t) <$ 129.94 km/h, where the higher limit comes from the word speed record achieved on skateboard in standing position. The lower limit is also necessary, since, the skaters drive the board with their legs at low speeds. The acceleration ($a_{\max} = 1.7383$ m/s^2) is determined based on the grade of the slope (18 %) where the speed record was set. Parameter k is chosen based on the measurement of Loram and Lakie [4]. The other parameter are as follows: $h = 0.85$ m, $m = 75$ kg, $g = 9.81$ m/s^2, $l = 0.3937$ m, $\kappa = 63°$, $b = 0.1$ m.

The semidiscretization method was used to determine the appropriate dimensionless gains (P, D) for any tested speeds, where the investigated speed domain was divided into 100 discrete parts. The dark grey shaded region in the plane (P, D) in Fig. 2.a is stable in frozen time sense.

During the numerical simulations, a pair (P, D) is called stable until the absolute body angle φ does not reach the 10° limit for any of the six following initial condition: (1) $\sigma(t) = 0$, $\varphi(t) = 0.01u(t)$; (2) $\sigma(t) = 0.01u(t)$, $\varphi(t) = 0$; (3) $\sigma(t) = 0.01u(t)$, $\varphi(t) = 0.01u(t)$; (4) $\sigma(t) = 0$, $\varphi(t) = 0.05$; (5) $\sigma(t) = 0.05$, $\varphi(t) = 0$; (6) $\sigma(t) = 0.05$, $\varphi(t) = 0.05$ for $t \leq 0$, where $u(t)$ is the Heaviside step function. The resulting finite time stable domain is shown by light gray shading in Fig. 2.a. This domain is larger than the one obtained by the frozen time method. According to the numerical simulations (see in Fig. 2.b), one can see, that the amplitude of the absolute body angle (φ) decreases in time for parameters taken from the dark gray shaded region. If the parameters are chosen from the light gray region then the amplitude increases in a time interval but it does not reach the limit 10°. If the parameters are taken from the white region, then the amplitude increases exponentially.

In addition, the frozen time stability analysis was produced for different accelerations between $-a_{\max} \leq a \leq a_{\max}$, on the same speed domain (not shown here). The area of the stable domain in the (P, D) parameter plane shows that the higher the acceleration, the easier the stabilization.

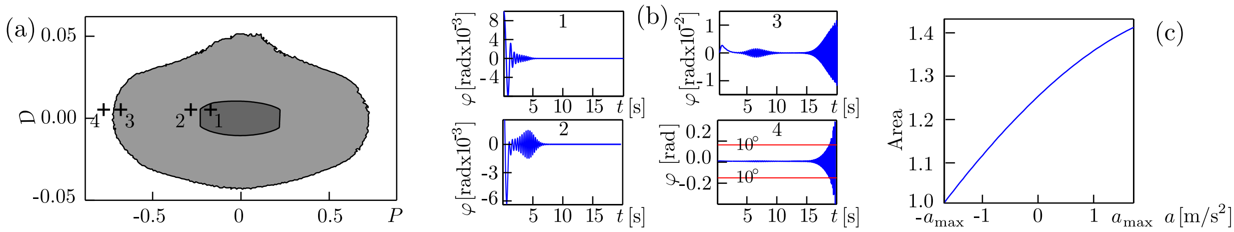

Figure 2: Stability analysis of accelerating skateboard: (a) piratical and frozen time stability charts, (b) numerical simulation results, (c) relative area of the frozen time stable domain respect to the acceleration.

Conclusions

A simple mechanical model of the accelerating skater-skateboard system was presented, where the describing equations are nonlinear time-varying neutral delay-differential equations. The conservative frozen time method was used, and the results were verified by means of numerical simulations. The finite time stability obtained by numerical simulations showed larger domain of permissible control gains is permissible to that of the frozen time method. This practical result however depends on the starting speed according to the sometimes increasing amplitude of the absolute body angle. For instance, it can happen that a control gain pair is called finite time stable if the starting speed is 5 km/h, but it will not be finite time stable anymore if the starting speed is 15 km/h. This result can explain the observed loss of stability phenomenon at higher speeds. Moreover, based on the investigation of the effect of the magnitude of the longitudinal acceleration, one can say that the higher the acceleration, the easier the stabilization.

References

[1] Hubbard M. Lateral dynamics and stability of the skateboard. Journal of Applied Mechanics. 1979;46:931–936. doi: 10.1115/1.3424680

[2] Hubbard M. Human control of the skateboard. Journal of Biomechanics. 1980;13(9):745–754. doi: 10.1016/0021-9290(80)90236-5

[3] Varszegi B., Takacs D., Stepan G., Hogan S. J.: Stabilizing skateboard speed-wobble with reflex delay, *Journal of the Royal Society Interface* **13**, 20160345, (2016) doi: 10.1098/rsif.2016.034

[4] Loram, I. D., Lakie, M. (2002). Direct measurement of human ankle stiffness during quiet standing: the intrinsic mechanical stiffness is insufficient for stability. *The Journal of Physiology*, 545(3), 1041-1053. doi: 10.1113/jphysiol.2002.025049