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# Optimization of Planetary Gear Systems

M. Barbieri, A. Masoumi, F. Pellicano
*Dipartimento di Ingegneria Enzo Ferrari, Univ. Modena e Reggio Emilia, Modena, Italy*

*Summary*. This study investigates the effect of tooth profile modification on single stage spur planetary gear vibrations. A nonlinear dynamic model of planetary gear system is used to study the effect of tooth profile modifications at sun–planet and ring–planet meshes on vibration behavior. In order to avoid modification on ring gear, both tip and root reliefs are considered for sun and planet gears. The static model for planetary gear system with tooth profile modification is validated through comparisons with a complete finite element analysis performed using a commercial software CALYX (Planetary2D).
In the present study, an optimization approach based on Genetic Algorithms (GA) is proposed to improve planetary gear dynamic performances toward vibration reduction. As proven by dynamic analyses, a genetic algorithm is an effective optimization tool to design reliable profile modifications for reducing the planetary gears vibration amplitude over a wide frequency range.

## Introduction

The most important source of vibration in planetary gears is the parametric excitation due to the periodically time-varying mesh stiffness of each sun-planet and ring-planet gear, because the number of tooth pairs in contact changes during gear rotation. This mesh stiffness variation parametrically excites the planetary gear system, causing severe vibrations when a harmonic component approaches one of the natural frequencies (or their linear combinations). Under certain near resonant operating conditions, gear systems can experience a teeth separation that induces nonlinear effects such as jump phenomena and sub-harmonic and super-harmonic resonances with dramatic effects on the dynamic response [1]. These phenomena have been deeply investigated in geared systems during the last 20 years [2-7].
The previously validated model for nonlinear dynamics of planetary gears using all degrees of freedom for sun, planets, carrier and ring gears [8] is investigated here by adding tooth profile modifications on different gear element of this case study (see Figs.1, 2).
GA is used to optimize the sun and planet's teeth profiles. The goal of this optimization approach is to find an optimal set of profile modifications in order to reduce the overall vibrations in planetary gears. The objective function is the peak to peak of the static transmission error of the planetary gear; therefore optimization is performed on a static basis and for the nominal load. The effectiveness of the optimized set in terms of broadband dynamic response will be checked by means of the dynamic model.

### Dynamical Model

A nonlinear 2D lumped mass model of planetary gear system with time varying mesh stiffness, bearing compliance and nonsmooth nonlinearity due to the backlash and tooth profile modification is taken into account. The time varying meshing stiffnesses and error functions are evaluated by means of a nonlinear finite element model, through an accurate evaluation of global and local tooth deformation.
The effect of tooth profile modification on mesh stiffnesses and contact losses due to the profile modifications are taken into account as well as backlash.

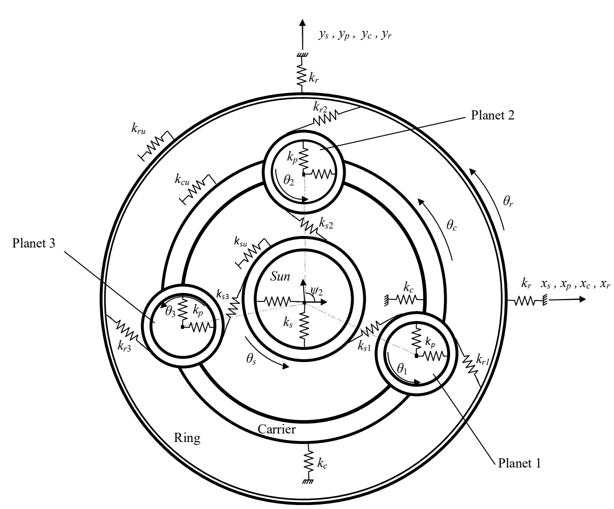


**Fig.1. Physical model of the single stage planetary gearbox with tooth profile modifications**

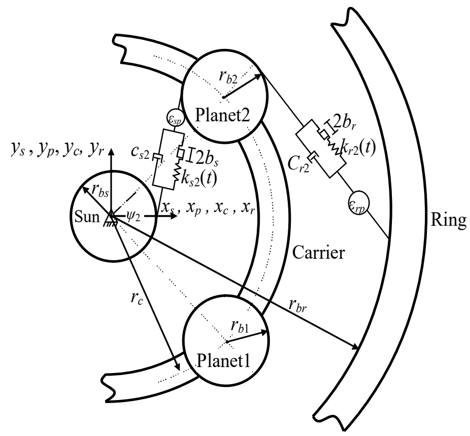


**Fig.2. Details of the meshing modeling with tooth profile modification**

The basic dynamical equilibrium equations contain (3$N$+9) nonlinear ordinary differential equations, where $N$ is the number of planets [8]. For the sake of brevity readers are referred to the ref. [8] to see all the equations of motion.

The effect of tooth profile modification on mesh stiffnesses and contact losses due to the profile modifications are taken into account as well as backlash. The error functions $\varepsilon_{sp}$ and $\varepsilon_{rp}$ used for modeling the teeth profile modifications for sun-planet and planet-ring meshes are periodically time varying at the mesh frequency similarly to $k_{sn}(t)$ and $k_{rn}(t)$; the mesh stiffnesses between sun-planet and ring-planet gears; and The excitation frequency $\omega_m$ is $\omega_m = \omega_s z_s z_r / (z_s + z_r)$, where $\omega_s$ is the angular velocity of the sun gear and $Z_s$ and $Z_r$ are the number of teeth for the sun and the ring gears.

Mesh stiffnesses and the error functions are evaluated by means of '*HelicalPair*' software [11]. These functions are evaluated separately for external gears (sun-planets mesh) and internal gears (ring-planets mesh).

The gears are in contact only if the relative motion of the mating gears along the line of action exceeds the gap of a tooth pair which is equal to the sum of profile modification amounts at both nominally contacting point plus total backlash.

Fig. 3(a) shows standard profile modifications on a spur gear tooth, which consist in a removal of material from the tip (tip relief) or the root (root relief), according to different manufacturing parameters. The ''start roll angle at tip'' *αts* and the ''magnitude at tip'' *magt* specify the point on the profile at which the relief starts and the amount of material removed at the tip radius; *αrs*, *magr* and αre have similar meaning, where the current roll angle α is given by $\alpha = \sqrt{\left((d / dg1)^{\wedge} 2 - 1\right)}$, see Fig. 3(a). Since the removal of material is measured along the direction normal to the profile, usual representations of the reliefs are given as deviation from the theoretical involute profile: Fig. 3(b)

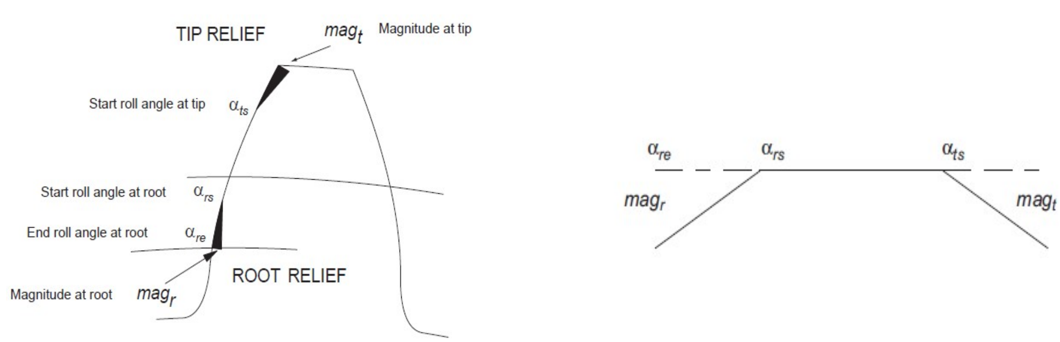


**Fig.3. a: Profile modification parameters (deviation from involute), b: k-chart along tip and root relief for linear modifications.**

**Optimization of a planetary gear system by means of GA**

Optimal set of profile modifications are found using genetic algorithm, for minimising the overall vibrations in planetary gear set. The objective function is the peak to peak of the static transmission error of the planetary gear. Optimization parameters are starting point and magnitude of tip and root profile modifications for sun and planet. Therefore eight parameters are to be found by means of GA. A description of the algorithm can be found on Ref. [10]. Since it is not affordable profile modifications on ring are not considered, nevertheless applying both tip and root reliefs on planets by itself, is effective on both STE of the sun-planet and planet-ring mesh.

The effectiveness of the optimized set in terms of broadband dynamic response will be checked by means of the dynamic model. Parameters of the planetary gear analyzed in this study are given in Table 1. A constant torque of 1130 Nm is applied on sun gear and the ring gear is fixed. Natural frequencies of the model before and after optimization are listed in Table 2.

**Table1. Parameters of the case study planetary gear set**

| Parameter | Sun | Planet | Ring | Carrier |
|---|---|---|---|---|
| Number of teeth | 27 | 35 | 99 | - |
| Module [mm] | | 2.8677 | 2.8677 | - |
| Pressure angle [deg] | | 24.60 | 20.19 | - |
| Working Center Distance [mm] | | 88.89 | 88.89 | - |
| Root diameter [mm] | 70.485 | 91.440 | 284.150 | - |
| Outer diameter [mm] | 84.074 | 105.004 | 304.800 | - |
| Inner diameter [mm] | 57.15 | 73.66 | 271.73 | - |
| Base diameter [mm] | 70.40 | 91.26 | 258.130 | 177.80 |
| Translational bearing stiffness [N/m] | 2.19e9 | 2.19e8 | 2.19e10 | 2.19e10 |
| Rotational bearing stiffness [N.m/rad] | 0 | 0 | 2.19e10 | 2.19e10 |
| $I/r^2$ [kg] | 1.56 | 2.46 | - | 24.80 |
| Mass [kg] | 1.64 | 1.33 | - | 21.82 |

Figures 4, 5 represent the comparison of RMS (mean removed) of sun and three planets rotation for the planetary gear sets with pure involute teeth and modified teeth. These figures show that the vibration levels obtained from optimized set are smaller than the vibration levels obtained using a pure involute. This confirms the good dynamic performance of the optimized gear set.

Figrues 5, 6 also show a perfect symmetry of the system before and after optimization, as long as the system geometry and loading are symmetric, the system response of the system remains symmetric as well, even after applying optimization and there is perfect force balance on planetary gear elements; this means sun bearings experience negligible forces (theoretically zero).

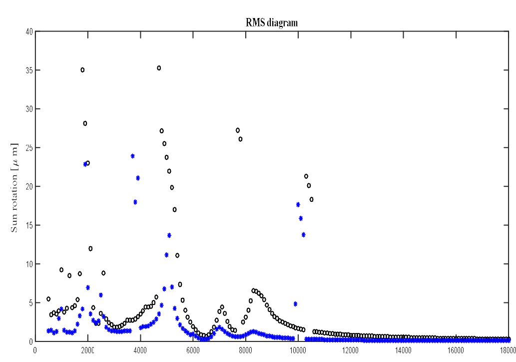


**Fig.4. RMS of sun rotation vs. mesh frequency; blue stars present the effect of optimization on planetary gear response, black circles results for pure involute gears**

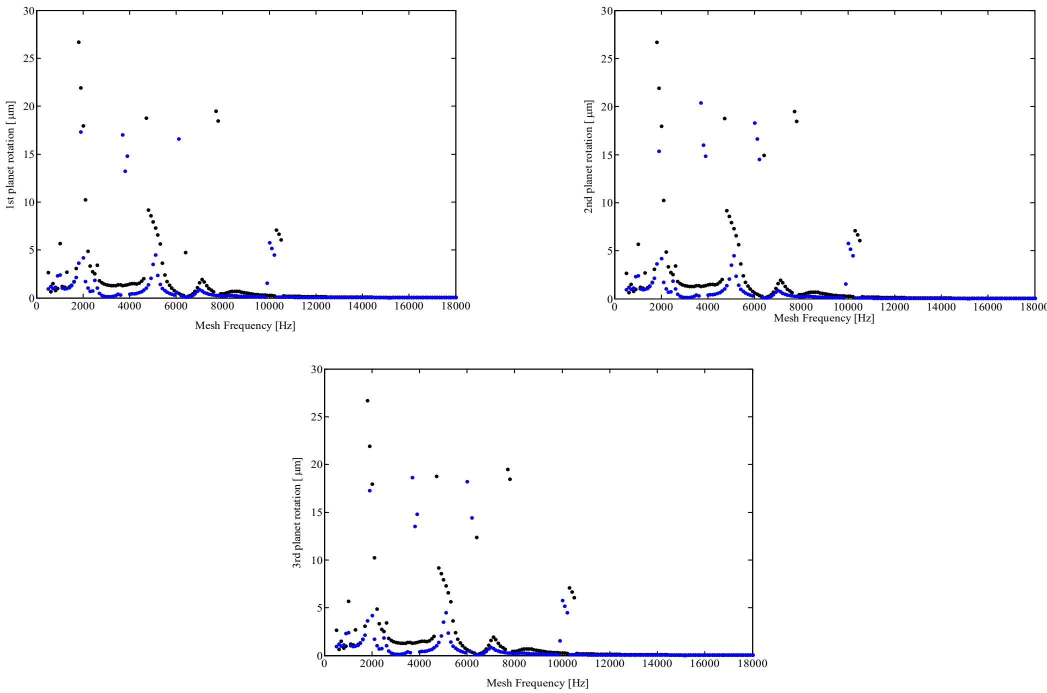


**Fig.5. RMS of planets rotational vibration (blue points: effect of optimization)**

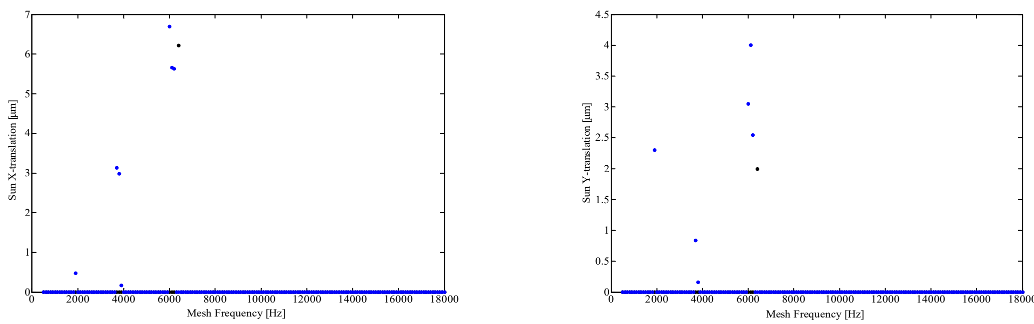


**Fig.6. sun X and Y translations vs. mesh frequency for (blue points: effect of optimization)**

The bifurcation diagrams are extracted by varying the mesh frequency. Fig. 7, 8 represent the bifurcation diagram of sun and planets rotation. In these figures results are in comparison with the previous results presented in Ref. [8] i.e. gears with pure involute profile. Blue points present the results of the optimized planetary gear after tooth profile modifications, and black points are results for pure involute gears.

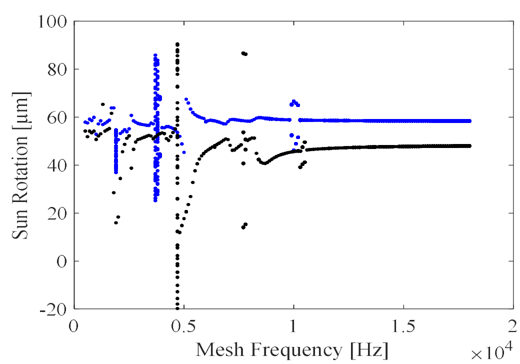


**Fig.7. Bifurcation diagram vs. mesh frequency for sun rotation; black points show the results without TPM, blue points show the results with TPM**

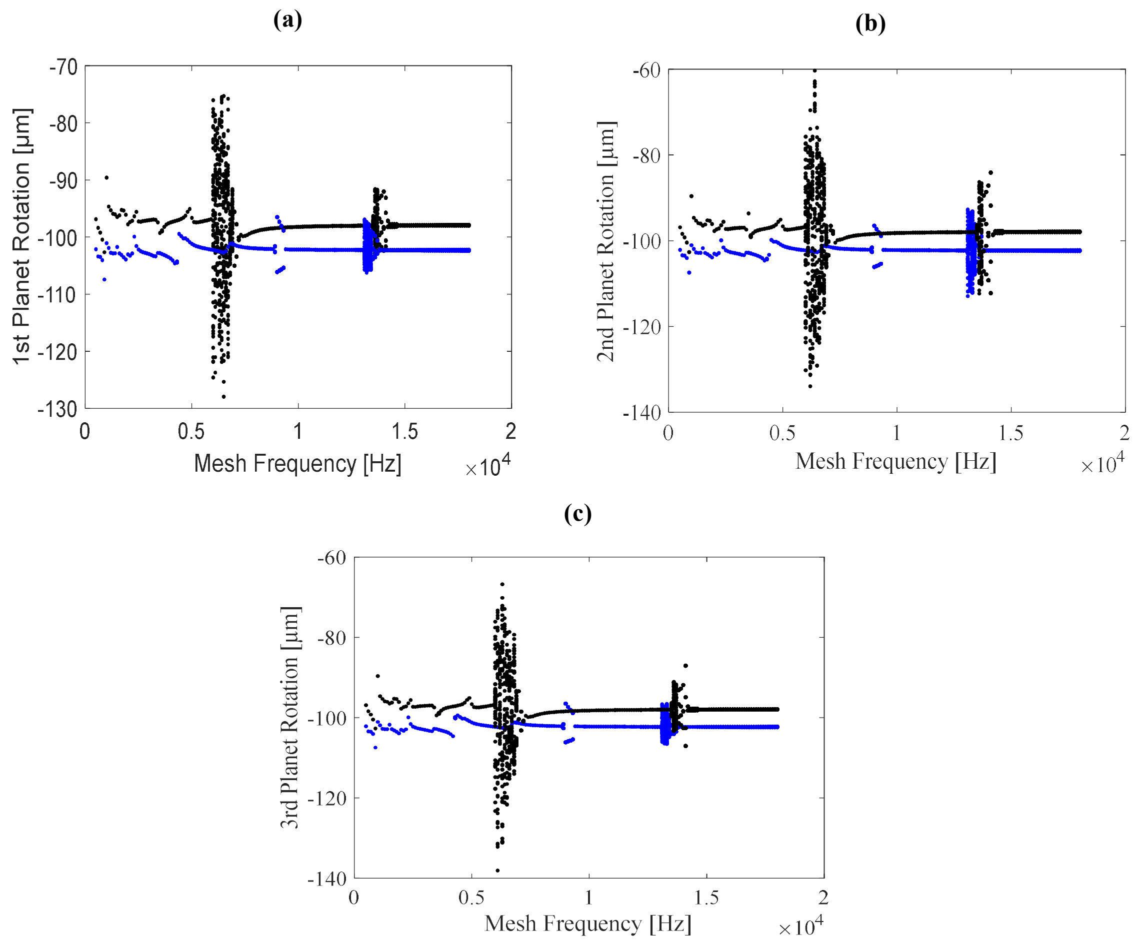


**Fig. 8. Bifurcation diagram vs. mesh frequency for case 2 (a) first planet rotation[µm], (b) second planet rotation [µm], (c)third planet rotation [µm] ; black points show the results without TPM, blue points show the results with TPM**

**Table 2: Comparison of natural frequencies for planetary gear model with and without TPM**

**(Planet translational bearing stiffness is equal to 2.19e9 N/m)**

| Frequency mode number | Natural frequency [Hz] case 1: *pure involute gear model* | Natural frequency [Hz] case 2: *optimized model* |
|---|---|---|
| 1 | 1749 | 1728 |
| 2 | 1749 | 1728 |
| 3 | 2017 | 1904 |
| 4 | 3253 | 3089 |
| 5 | 3253 | 3089 |
| 6 | 5188 | 5030 |
| 7 | 6373 | 6346 |
| 8 | 6373 | 6346 |
| 9 | 7096 | 6960 |
| 10 | 7334 | 7221 |
| 11 | 7334 | 7221 |
| 12 | 8451 | 8220 |
| 13 | 8507 | 8328 |
| 14 | 8507 | 8327 |
| 15 | 22295 | 22198 |
| 16 | 22295 | 22198 |
| 17 | 53264 | 53264 |
| 18 | 149096 | 149074 |

Profile modifications are effective in the frequency ranges where no contact loss occurs; and in the case of contact loss, optimal profile modifications reduce the ranges. Tooth profile modification is also effective on meshing stiffnesses and natural frequencies of the system. The natural frequencies of both models (before and after optimization) are listed in Table 2. (Mean values for sun-planet mesh stiffnesses for this case study before and after applying the tooth profile modifications are respectively equal to 6.14e+08 N/m and 5.59e+08 N/m).

## Conclusions

The effect of tooth profile modifications on vibration reduction of sun and planet gears is studied in this work. The GA is used to find the optimal profile modifications on the sun and planets and the dynamic behavior of the entire model of the planetary gear system is compared at different rotational velocities before and after applying tooth profile modifications on sun and planet gears.

The present study shows the effect of tooth profile modifications in presence of backlash. The mesh stiffnesses are calculated for gear meshes before and after applying tooth profile modifications and the effect of tooth profile modifications on natural frequencies of the system is obvious in results of dynamic analyses.

In order to evaluate the effectiveness of the proposed method (using GA) for optimization of planetary gear system, dynamic analyses are performed and the results show that optimization of the system can greatly reduce the overall vibrations.

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