Passive/active thermal dynamics in the coupled nonlinear vibrations of laminated plates

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Summary. A minimal reduced order model of von Karman shear indeformable laminate with assigned linear temperature variation along the thickness is used to investigate the passive or active role played by the thermal dynamics in the nonlinear thermomechanically coupled vibrations of an orthotropic plate. Meaningful effects induced on the system response by variations of thermal properties are highlighted, along with the possible inadequacy of an uncoupled mechanical model to grasp the system actual dynamics.

Introduction

Thermoelastic analysis of composite plates and shells is of interest in mechanical, aerospace and civil engineering applications. Partial or full thermomechanical coupling has been addressed through various models and mostly finite element-based approaches in statics as well as linear or nonlinear dynamics [1-4]. In contrast, few works have dealt with thermomechanical coupling in a Reduced Order Model (ROM) perspective [5,6]. In the framework of the 2D modelling of geometrically nonlinear laminated plates, unified continuous and discretized formulations of the thermomechanical problem for a von Karman plate with assigned linear temperature variation along the thickness has allowed to end up to a coupled three-mode (one mechanical and two thermal) ROM [7], useful to get information on some basic nonlinear dynamic phenomena induced by full/partial thermomechanical coupling. Later, in the context of a comparison of different models [8], a refined von Karman plate with third-order shear deformability and consistent cubic variation of the temperature along the thickness has been formulated, ending up, under certain conditions, to the same minimal model, though more refined because of accounting for a larger variety of possible thermal excitations

Nonlinear dynamics of a thermomechanically coupled model

The afore-mentioned modelling framework [7,8] is here used to investigate the role played by the thermal variables of the coupled problem in the presence of either a solely mechanical excitation, which activates a merely passive thermal dynamics, or its combination with also a thermal excitation, which makes the active role of the thermal dynamics apparent. To this aim, referring to a simply supported orthotropic square plate with movable and isothermal edges, subjected to an in-plane pretension load p and a transverse harmonic excitation of amplitude f, along with a possible free heat exchange on the upper and lower surfaces due to a T_∞ constant difference of temperature between the plate and the surrounding medium, the minimal ROM of the classical von Karman, shear indeformable, plate with assumed linear temperature variation over the thickness can be used, which reads [7]:

$$\ddot{W}(t)+a_{12}\dot{W}(t)+(a_{13}+a_{14}p)W(t)+a_{15}W(t)^3+a_{16}T_{R1}(t)+a_{17}T_{R0}(t)W(t)+a_{18}f\;Cos(t)=0$$

$$\dot{T}_{R0}(t)+a_{22}T_{R0}(t)+a_{23}T_\infty+a_{24}W(t)\dot{W}(t)=0$$

$$\dot{T}_{R1}(t)+a_{32}T_{R1}(t)+a_{33}\dot{W}(t)=0$$

The nondimensional equations describe the dynamics of the midplane transverse deflection (W), and of the membrane (T_{R0}) and bending (T_{R1}) temperatures, respectively; a_{ij} are coefficients which incorporate the geometrical and physical properties of the model. Bifurcation diagrams, behavior charts and basins of attraction have been obtained to comprehensively understand the dynamical effects of some system parameters and of a possible thermal variation. As regards the system with sole mechanical excitation (i.e. with $T_\infty = 0$), besides verifying the important role of the mechanical (pretension and transverse) forcing parameters, the results highlight the decisive role played by some intrinsic parameters in modifying the system response. In particular, the thermal expansion α_2 (included in some a_{ij} coefficient), which is one of the thermal properties characterizing the plate material, is seen to have a substantial effect in regularizing the system nonlinear dynamics, as shown in Fig. 1(a). Indeed, the evident multistability (i.e. five 1-period and two 4-period solutions) which characterizes the post-buckling configuration for the default value $\alpha_2 = -1.91\cdot 10^{-6}$ is strongly reduced as α_2 increases, up to the survival of a single pair of buckled solutions for high values of the parameter. The relevant basins of attraction (herein not reported) underline a regularizing effect of the parameter also in the organization of the mechanical phase plane, with the marked fractality related to the multistable condition being strongly reduced as the thermal expansion increases, and with an increasingly clear separation of the initial conditions leading to the various attractors.

When also introducing an external thermal variation which activates pure thermal convection on the external surfaces and pure internal thermal conduction, it is particularly interesting to evaluate the influence of the thermomechanical coupling on the system dynamical behavior, not only in terms of transient evolution of the response, but also as

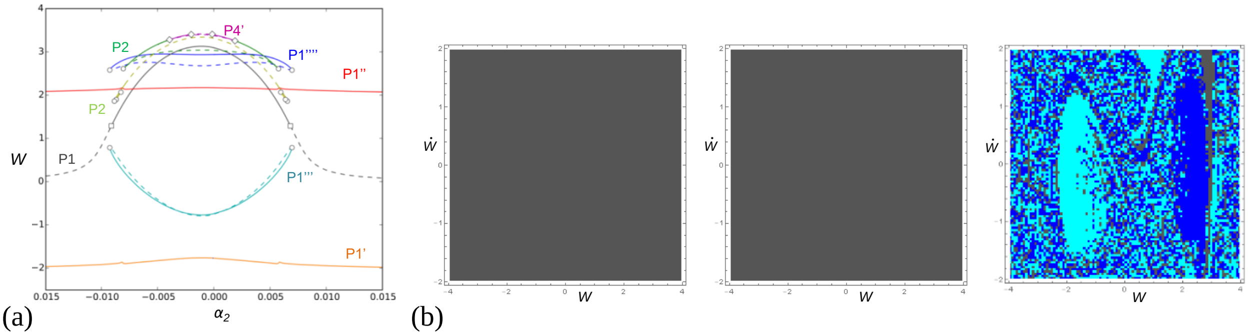

Figure 1: (a) Reducing multistability through variation of thermal expansion. (b) Cross-sections of 4D basins of attraction with coupled model (monostable pre-buckled response without (left) and with (middle) assigned transient thermal variation) or uncoupled model (buckled responses induced by assigned steady thermal variation (right))

regards the stationary regime. In this respect, the possibility to obtain buckled solutions by exploiting the thermal variation between plate and surrounding environment, represented by the T_∞ parameter directly affecting the dynamics of the membrane temperature, has been investigated. Some main outcomes are summarized in Fig. 1(b), where three 2D cross-sections (corresponding to trivial thermal initial conditions, i.e. $T_{R0}(0)=T_{R1}(0)=0$) of the 4D basins of attraction are reported. The left-hand mechanical phase plane (gray basin) refers to the monostable solution obtained for a pre-buckling pretension value. Starting from that, a thermal $T_\infty=100$ variation is added to the model, thus activating the membrane temperature variable which rises up to the steady state mean value $T_{R0}\cong 0.9$. The relevant coupling term in the mechanical equation modifies the mechanical stiffness and brings the system to a multistable scenario with the arise of a pair of buckled solutions. These are visible (blue/cyan basins) in the right-hand phase plane of Fig. 1(b), which represents the response of the uncoupled mechanical system (the sole first equation with $a_{16}=0$) with the stiffness modified by the assigned steady value $T_{R0}\cong 0.9$. However, the response of the actually coupled system displays a dramatically different scenario characterized by a total dominance of the pre-buckling response, as shown by the middle phase plane of Fig. 1(b). This is due to the long transient time needed by the membrane temperature to attain its final steady value, which provides its contribution to the mechanical stiffness very slowly. As a consequence, the mechanical response falls back on the pre-buckling solution, which remains stable also after the arise of the steady buckled responses. It is worth noting that the discrepancies between expected and actual behaviors of the model can be eliminated by properly acting on the thermal stiffness term in the second equation, whose a_{22} coefficient can be varied in such a way to reduce the T_{R0} settling time necessary to bring the dynamical response to the buckled configurations.

Conclusions

Dynamical analyses accomplished in the absence of thermal excitations have highlighted the important role played by some physical properties of the material in strongly modifying the system response, which makes them useful as control parameters in a design/operational perspective. In turn, the addition of thermal changes has pointed out the critical effect of the transient coupled thermal dynamics on also the mechanical steady behavior, emphasizing the possible inadequacy of an uncoupled, though thermally affected, mechanical ROM in grasping the system actual response, and the need of comprehensive dynamical analyses allowing to understand the rich scenarios of actually coupled models.

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