Nonlinear Random Vibrations of Stretched Beam Discretized by Finite Difference Scheme and Excited by Gaussian White Noise

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Summary. With finite difference method, a multi-degree-of-freedom nonlinear stochastic dynamical system is formulated by discretizing the stretched Euler-Bernoulli beam being of pin-supported ends and excited by uniformly distributed Gaussian white noise. The probabilistic solutions of this system are studied by state-space-split (SSS) method and exponential-polynomial closure (EPC) method to examine the effectiveness and efficiency of this solution procedure. Numerical results show that the probabilistic solutions obtained by SSS-EPC method agree well with Monte Carlo simulation. The computational efficiency of SSS-EPC method is much higher than that of Monte Carlo simulation. Compared to the probabilistic solutions obtained by equivalent linerization method, the results obtained by SSS-EPC method are much improved for the studied strongly nonlinear system.

Introduction

The stretched beam can find its applications in science and engineering. There are a lot of publications about the vibrations of the stretched beam. However, there are only few studies about the random vibrations of this beam, particularly the study on the probabilistic solution of the stretched beam [1, 2]. The random vibration of the stretched beam can find its applications in the area of seismic analysis and structural reliability analysis. When the beam is discretized by finite difference scheme for numerical analysis, the formulated system is a multi-degree-of-freedom (MDOF) nonlinear stochastic dynamical (NSD) system for random vibration analysis. Many real problems can be described as MDOF-NSD systems. Excited by Gaussian white noise, the probability density function (PDF) of the responses of NSD system is governed by Fokker-Planck-Kolmogorov (FPK) equation which exact solution is not obtainable for MDOF-NSD systems. Equivalent linearization method (EQL) was frequently employed to obtain the approximate means and second moments of the responses of weakly nonlinear systems [3]. Monte Carlo simulation (MCS) is another method that is applicable for the numerical solution of MDOF-NSD systems, but the computational efficiency, numerical stability, convergence, round-off error, and requirement for large sample size can be challenges with MCS in analyzing large systems [4, 5]. The state-space-split (SSS) method was proposed for the approximate solutions of the high-dimensional FPK equations [6, 7]. By the SSS method, the high-dimensional FPK equation is reduced to low-dimensional FPK equations which can be further solved by the exponential polynomial closure (EPC) method [8]. The whole solution procedure is refereed to as SSS-EPC method. In this paper, the SSS-EPC method is applied to analyzing the probabilistic solutions of the stretched Euler-Bernoulli beam excited by uniformly distributed Gaussian white noise. The MDOF-NSD system is formulated by discretizing the stretched beam with finite difference scheme. The objective of this study is to examine the effectiveness and efficiency of the SSS-EPC method in analyzing the probabilistic solutions of the presented beam system.

Probabilistic Solutions of Stretched Beam

Consider the Euler-Bernoulli beam with pin supports at its two ends and excited by uniformly distributed Gaussian white noise as shown in Fig. (1). The governing equation of this beam is

$$\rho\ddot{Y}(x,t)+\dot{Y}(x,t)+EIY^{(4)}(x,t)-\frac{EA}{2L}Y''(x,t)\int_0^L Y'^2(x,t)dx=qW(t) \tag{1}$$

where $Y(x,t)$ is the deflection of the beam at time t at the location with distance x from the left-hand side of the beam; ρ is the mass density of the material; c is the damping constant; E is the Young's modulus of the beam material; I is the moment inertia of the cross section of the beam; A is the area of the cross section of the beam; L is the length of the beam; $qW(t)$ is the uniformly distributed loading laterally applied on the beam, q is a constant and $W(t)$ is Gaussian white noise with power spectral density S. With finite difference scheme as shown in Fig. (1), Eq. (1) can be discretized into the following system.

$$\begin{aligned}&\ddot{Y}_n+\frac{c}{\rho}\dot{Y}_n+\alpha(Y_{n+2}-4Y_{n+1}+6Y_n-4Y_{n-1}+Y_{n-2})-\beta(Y_{n+1}-2Y_n+Y_{n-1})\sum_{i=1}^{N+1}(Y_{i+1}^2+Y_i^2+Y_{i-1}^2\\&+Y_{i-2}^2+Y_{i+1}Y_i-2Y_{i+1}Y_{i-1}-Y_{i+1}Y_{i-2}-Y_iY_{i-1}-2Y_iY_{i-2}+Y_{i-1}Y_{i-2})=\frac{q}{\rho}W(t)\qquad(n=1,2,...,N)\end{aligned} \tag{2}$$

where $\alpha=\frac{EI}{h^4\rho}$ and $\beta=\frac{EA}{24Lh^3\rho}$. The boundary conditions are $Y_0'=0$ and $Y_{N+1}'=0$, which gives $Y_0=0, Y_N=0, Y_{-1}=-Y_1$ and $Y_{N+2}=-Y_N$. Eq. (2) is a MDOF-NSD system excited by Gaussian white noise. Give $L=5m$, $E=2.1\times10^{11}pa$, $I=2.17\times10^{-4}m^4$, $A=8.6\times10^{-3}m^2$, $\rho=7.85\times10^3kg/m^3$, $c=10^3$, and $qW(t)=10^4W(t)N/m$ with $S=5$. The number of unknowns N in finite difference scheme is set to be 7. Based on Eq. (2), the PDFs of the deflection in the middle of the beam $Y(0.5L,t)$ (or Y_4) and velocity $\dot{Y}(0.5L,t)$ (or $\dot{Y}_4$) are analyzed by SSS-EPC method when the polynomial degree n equals 4 in the EPC procedure. The PDFs and logarithm of PDFs of Y_4 and $\dot{Y}_4$ obtained by SSS-EPC, MCS, and EQL,

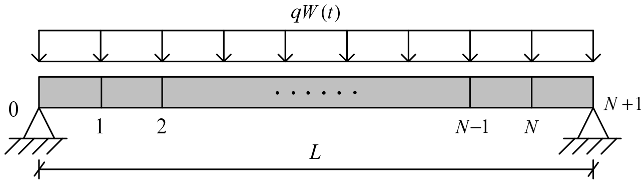
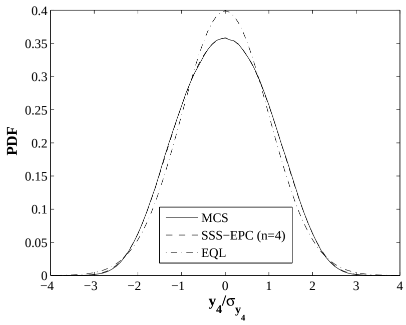
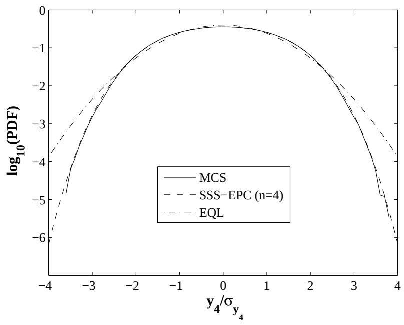


Figure 1: Finite difference model of the Euler-Bernoulli beam with pin supports at its two ends and excited by uniformly distributed Gaussian white noise

respectively, are shown and compared in Figs. 2(a) and 2(b), respectively. The number of samples used in MCS is 10^8. It is seen from the numerical figures that the results obtained by SSS-EPC method are close to MCS while those obtained by EQL deviate a lot from MCS. Under the same computer running environment, the computational time needed by MCS is about 1,000 times of that needed by SSS-EPC method for this 7-DOF system. The value of this ratio can increase rapidly as the number of system degrees of freedom increases.



(a) Frequency response curve with $f = 1000$



(b) Time history with $\Omega = 15$ rad/s

Figure 2: a) PDFs of displacement in the middle of the beam; (b) Logarithm of PDFs of displacement in the middle of the beam.

Conclusions

The SSS-EPC method is studied in analyzing the probability density functions of responses of the stretched Euler-Bernoulli beam. The MDOF-NSD system about the nonlinear random vibration of the beam is formulated by finite difference scheme. Numerical results are presented about the probabilistic solutions of the beam with pin supports at its two ends and excited by uniformly distributed Gaussian white noise which is fully correlated in space. By SSS-EPC method, the obtained PDFs of the deflection of the beam are close to MCS even in the tails of the PDF solutions. The numerical analysis shows that the SSS-EPC method works for accurately and efficiently analyzing the probabilistic solutions of the stretched Euler-Bernoulli beam excited by uniformly distributed Gaussian white noise when the MDOF-NSD system governing the nonlinear random vibration of the stretched beam is formulated with finite difference scheme.

Acknowledgement

The results presented in this paper were obtained under the supports of the Research Committee of University of Macau (Grant No. MYRG2014-00084-FST).

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