DYNAMICS OF STATICALLY PRE-LOADED HUMAN AORTA WITH RESIDUAL STRESSES

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Summary The response of human aortic segment under static and dynamic pressure is investigated. The aortic segment is modelled as three-layered hyperelastic and viscoelastic circular cylindrical shell with pre-stress. Both geometrical and physical nonlinearities are taken into account. The effect of internal quiescent fluid and pulsatile pressure is also taken into account. The results are presented in the form of pressure-deflection curves and frequency responses to pressure excitation.

PROBLEM DESCRIPTION

Static and dynamic analysis of circular cylindrical shell that models a segment of human aorta is carried out in this study. The shell is assumed to be three-layered, with each layer being hyperelastic, fiber-reinforced, viscoelastic material. Following the paper [1], we chose a material model in form of combination of Neo-Hookean and Fung models. The strain energy density for this case has the following form:

$$W = W_{NH} + W_F, \tag{1a}$$

$$W_{NH} = \frac{E}{6}(I_1 - 3), \tag{1b}$$

$$W_F = C(Exp(c_{11}\varepsilon_x^2 + c_{12}\varepsilon_x\varepsilon_\theta + c_{22}\varepsilon_\theta^2) - 1), \tag{1c}$$

where E is the Young's modulus of the shell material; I_1 is the first invariant of the right Cauchy-Green deformation tensor $\mathbf{C} = \mathbf{2E} + \mathbf{I}$, and $\mathbf{E}$ is Green-Lagrange strain tensor. The material (1) is anisotropic material due to Fung term, which is included to describe anisotropic response of the fibers and their sharp increase in stiffness. Neo-Hookean term describes the isotropic part of the tissue response. The geometrical nonlinearity is described with the higher-order shear deformation theory [2]. Residual stresses are taken into account.

The viscoelasticity is taken into account through the quasi-linear viscoelasticity developed by Fung [3].
The shell is filled with quiescent fluid [4]. Since we deal with large deflections and large strains, the displacement-dependent expression for pressure is used [5]. Such load is proportional to the area of the deformed surface and is orthogonal to it.

Strain energy density in the form (1) is not polynomial in strain components, which significantly complicates the investigation of the dynamics. The analysis is simplified by expansion of strain energy density into truncated after order 4 series in strain components. Such model is able to describe the behavior of the vessel only locally, in the vicinity of a given configuration of interest around which the expansion is made. To reach highly deformed configurations, a sequence of successive local models has to be built.

NUMERICAL EXAMPLE

We consider the shell with geometrical parameters that are similar to the parameters of the straight thoracic aortic segment of human aorta. The geometrical parameters are [6]: radius $R = 0.01575$ m, length $L = 0.126$ m. Thicknesses for three layers are: for intima, the internal layer, $h_i = 0.00033$ m, for media, the medium layer, $h_m = 0.00132$ m, for adventitia, the external layer, $h_a = 0.00096$ m. The material parameters are partially determined based on experimental data from [1], partially adopted from [1]. It is assumed that all three layers have the same density $\rho = 1200$ kg/m^3, the contained fluid density is taken $\rho_F = 1060$ kg/m^3.

The shell is simply supported at the ends. The boundary conditions, including the natural nonlinear boundary condition of zero stress resultant, are identically satisfied by the trigonometric functions of special form as shown in [6]. The convergence study has been carried out and it is concluded that the 29 degrees-of-freedom system provides good approximation of the solution.

Initially, the static problem of inflation of the shell under the pressure load was studied. The pressure-defection curve is shown in Fig. 1. As can be seen, the stiffness drastically increases after certain strain threshold is reached.

Afterwards we analyzed free and forced vibrations around pre-loaded state of the entire artery as a composite shell and also single-layered shell to understand the contribution of different layers. For the three-layered artery model the configuration in the middle of physiological range (12.5 kPa pressure) was chosen for the expansion of the strain energy density around it. Vibrations with dominant first axisymmetric bending mode and with dominant mode with two half-waves in circumferential direction of the empty and fluid-filled shells are studied. The frequency response for the empty shell

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made of adventitial material is shown in Fig. 2. We can see that the nonlinearity is weak enough for this case, however, in the case of fluid-filled shell it is stronger.

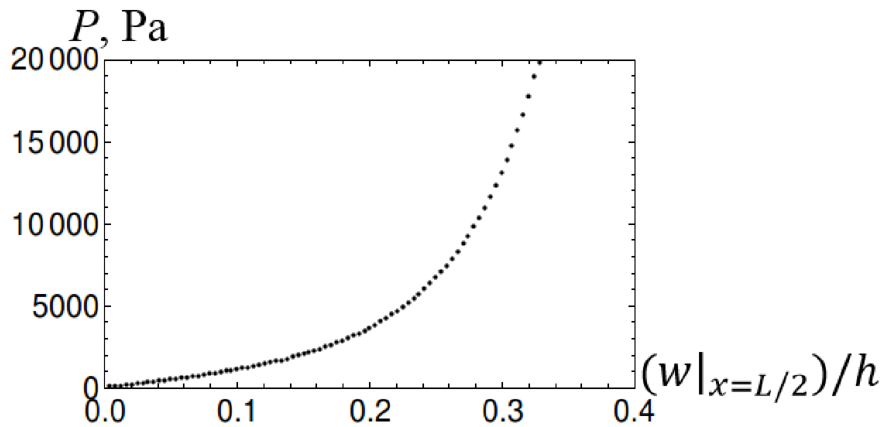

Figure 1. Static pressure-deflection curve. The abscissa shows central deflection.

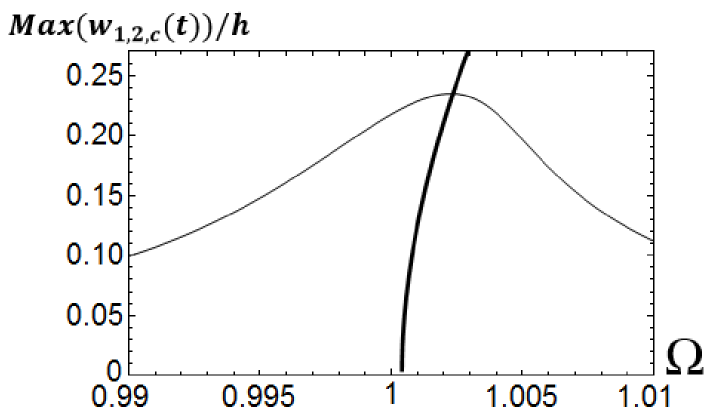

Figure 2. Frequency response of empty shell vibrations around pre-loaded configuration (adventitial material). Principal bending mode amplitude vs. frequency. Frequency response (thin line) and backbone curve (thick line).

CONCLUSIONS

The method for the analysis of nonlinear vibrations of arteries is presented. The method is able to deal with hyperelastic, viscoelastic, multilayered, fiber-reinforced shells that model blood vessels. The method is tested on the human aortic segment model. Under the static pressure initially soft shell becomes much stiffer with the pressure growth, which is the common feature for soft biological tissues. The dynamic nonlinearity is not very strong, but still nonlinear effects, like change in stiffening type, are observed.

References

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