Entrainment and Bifurcation Dynamics of a Dry Friction Oscillator

Charles Jacob, Bipin Balaram and B.Santhosh
Department of Mechanical Engineering, Amrita School of Engineering, Coimbatore, Amrita Vishwa Vidyapeetham, Amrita University, India 641112

Summary. In this work, the entrainment phenomenon in a discontinuous oscillator is investigated. The system considered is a single degree of freedom dry friction oscillator in the form of a mass on a moving belt. The harmonically excited oscillator is modeled as a Filippov system and the dynamics is investigated by switch model based numerical integration. Bifurcation diagrams are generated with excitation frequency as the parameter for different values of forcing amplitudes. Bifurcation diagrams show the existence of entrained periodic solutions separated by quasiperiodic windows. Increase in the value of forcing amplitude lead to increase in the bandwidth of entrained periodic solutions. It is also observed that in the entrained periodic regimes, the system exhibit discontinuity induced bifurcations.

Introduction

Entrainment or frequency locking in nonlinear systems with continuous nonlinearities such as externally excited Van der Pol oscillator has been investigated extensively in the literature [1]. During entrainment, the oscillator regains its periodicity which was lost by the external excitation with magnitude less than a critical value. The parameter regions for which the system is locked to the driving frequency can be understood from the Arnold tongue. The literature on entrainment phenomena in discontinuous oscillators is limited [2]. Systems with friction belong to the category of discontinuous systems as the state variables representing the system dynamics are confined to different subspaces at different instants of time. In this work, the frequency entrainment phenomenon in the harmonically excited single degree of freedom (sdf) dry friction oscillator is investigated for different values of excitation amplitudes. The model considered in this paper can be used to model stick-slip oscillation in brakes and clutches. The study will help to understand the entrainment phenomenon in systems undergoing stick-slip oscillations.

Model and Equations of Motion

The model shown in Fig.1(a) is a harmonically excited linear sdf oscillator resting on a belt moving with a constant velocity V_b. The non-dimensional equations of motion governing the dynamics of the system considering the Stribeck

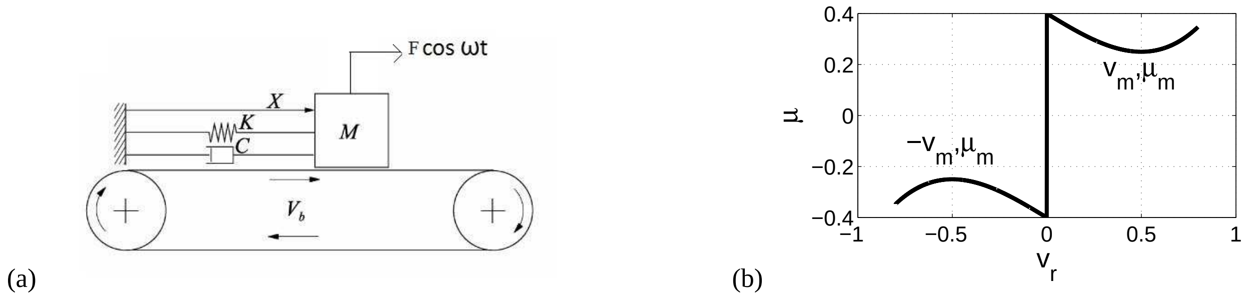

Figure 1: (a)Model of a discontinuous oscillator (b) Stribeck friction model

friction model shown in Fig.1(b) is given by

$$\ddot{x} + 2\beta\dot{x} + x + \mu_s\ sgn(v_r) - k_1 v_r + k_3 v_r^3 = f_0 \cos\omega t \tag{1}$$

with non-dimensional parameters $x = \frac{X}{L}, \beta = \frac{C}{\sqrt{KL}}$ and $f_0 = \frac{F}{N}$, where L is a characteristic length and N is the normal load between the mass and the belt. The constants $k_1 = \frac{3(\mu_s - \mu_m)}{2v_m}$ and $k_3 = \frac{(\mu_s-\mu_m)}{2v_m^3}$ where μ_s is the static friction coefficient, μ_m is minimum value of kinetic friction coefficient with v_m as the corresponding value of v_r, the relative velocity $\dot{x} - v_b$ and sgn represents the signum function. Eq.(1) is expressed as a Filippov system with $F_1(\mathbf{x})$ and $F_2(\mathbf{x})$ as the two vector fields for $v_r > 0$ and $v_r < 0$ respectively. For $v_r = 0$, the vector field is given by a convex combination of F_1 and F_2 as $F_{12} = F_1 + (1-\lambda)F_2$ [3].

Results and Discussions

The dynamics of the system is investigated with parameter values $v_m = 0.5, \mu_m = 0.25, \mu_s = 0.4, \beta = 0.05$. The belt speed v_b is assumed as 0.3. When $f_0 = 0$, the system exhibit self excited oscillation with an oscillation frequency of 0.9756 for the above given parameter values. Thomsen et al.[4] classified the motion of the above system based on the belt velocity as stick-slip oscillation, pure slip oscillation and steady sliding. In this paper, the analysis is limited in the region of stick-slip oscillation. The value of v_b is selected based on the above consideration.

The Filippov system is integrated numerically employing the switch model representation [5] and the bifurcation diagram with ω as the parameter with $f_0 = 0.1$ is generated and is shown in Fig.2(a). The bifurcation diagram shows large

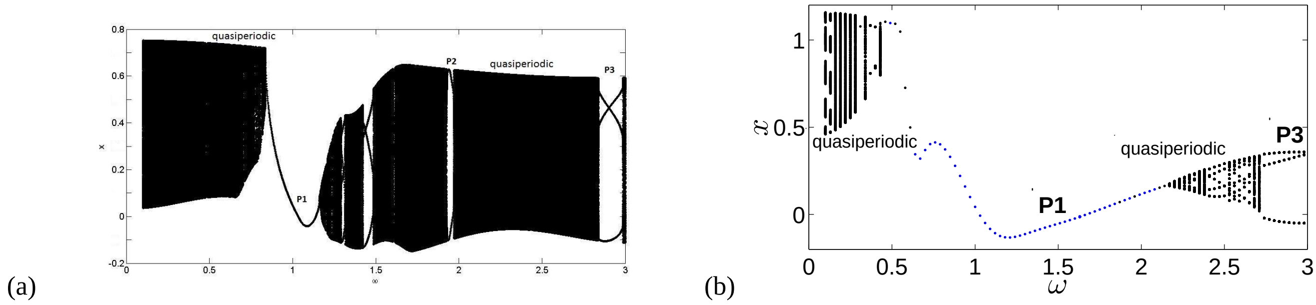

Figure 2: Bifurcation diagram with ω as the parameter (a) $f_0 = 0.1$ (b) $f_0 = 0.5$

regimes of quasiperiodic solutions and small windows of periodic solutions. It can be observed that close to the limit cycle frequency of $\omega_n = 0.9756$, the solution is periodic with period 1 (P1) and it is entrained ($1 : 1$ entrainment). This occurs not only at the value of 0.9756 but in a band of frequencies close to the above value. Similar phenomenon can be observed at integer multiples of ω_n as marked by **P2** and **P3** in the figure ($1 : 2$ and $1 : 3$ entrainment). The phase plane plots for different values of ω are shown in Fig.3(a)-(c). The bifurcation diagram with ω as the parameter is generated

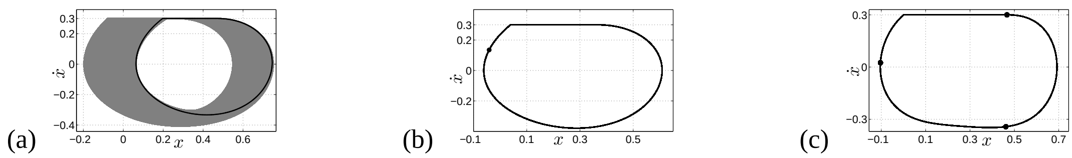

Figure 3: (a) Quasiperiodic solution for $\omega = 0.34$ (b) P1 solution for $\omega = 0.97$ (c) P3 solution for $\omega = 2.9$.

for $f_0 = 0.5$ and is shown in Fig.2(b). On comparison with Fig.2(a),the width of the quasiperiodic solution window get reduced and the range over which the $1 : 1$ entrainment takes place got increased. The $1 : 2$ entrainment window disappears whereas the $1 : 3$ entrainment window still exist. Another important observation is that the P1 solution exhibit discontinuity induced bifurcations (DIB) [3] such as adding sliding and switching sliding bifurcations. The phase plane plots corresponding to these bifurcations are shown in Figs. 4(a)-(c).

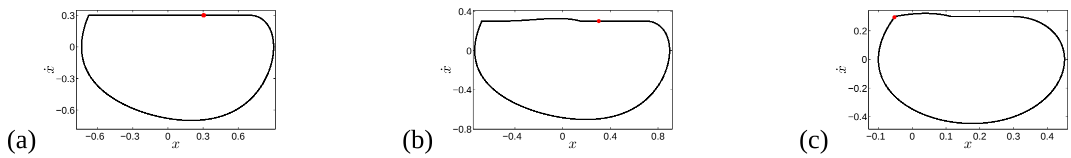

Figure 4: Phase plane plots (a) stick-slip $\omega = 0.65$ (b) adding sliding $\omega = 0.66$ (c) switching sliding $\omega = 1.5$

Conclusions

In this work, the entrainment phenomenon in a discontinuous oscillator is investigated by modeling it as a Filippov system. The bifurcation diagrams generated with ω as the parameter revealed that entrainment takes place at integer multiples of the limit cycle frequency for lower values of excitation amplitude. The periodic entrainment regions are separated by quasiperiodic windows. For an increase in the value of excitation amplitude, the regions of quasiperiodic solution shrinks and solution become entrained in a wider band of excitation frequency. This scenario is important in practical systems such as brake to improve the efficiency by reducing the sliding effect.

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