# Experimental Nonlinear Phenomena in Structures with Multiple Equilibria Controlled by Boundary Displacements: Ultra-fast Decay of Coupled Vibrations

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*Summary*. In our experimental structural dynamics studies, we discover that physical beam structures with displacement controlled hinged endpoints create a three-dimensional nonlinear dynamics setting supporting nonlinear phenomena with remarkable characteristics. In particular, whenever the hinged end points of a thin steel beam are fixed at a distance less than the beam length, all free local and global vibrations (sampled by state-of-the-art sensors) decay very fast towards one of the stable equilibrium states. All sampled free vibrations possess continuous, broadband frequency spectra, a fact in full contract to the discrete spectra characterizing free vibrations of cantilevered beams with a single equilibrium state. We conjecture that the ultra-fast dissipation of mechanical energy is the result of a scattering mechanism generated by the presence of multiple equilibrium states in an infinite space of vector field dynamics underlined by nonlinear cross-coupling among stretching, bending, and shearing modes of motion.

## Introduction

It is well-known that nonlinear oscillators-stemming as reduced order models for continua with multiple equilibrium states-can support complex nonlinear dynamics (chaos) [1,3]. Also global geometric objects in phase space in the form of invariant manifolds precipitated via interactions complicated dynamics. An interesting class of invariant manifolds used in the mathematical model reduction is that of slow and fast invariant manifolds. These geometric objects allow a decomposition of the domain and thus offering a tool for analyzing coupled dynamical systems. The bifurcation of slow and fast invariant manifolds of motions can be exploited for desired modification of the dynamics of linear continua by coupling a non-linear continuum with multiple equilibrium states [4,5]. The dynamics of complex structures may be manipulated by inserting dynamically multiple equilibria. Within this long time well-cultivated spirit of research in coupled dynamics, we experiment with physical beams to discover unknown nonlinear phenomena-with interesting energy dissipation characteristics-in continuum structural systems. By controlling the distance of the endpoints of a thin beam, we both create multiple equilibrium states and enable at the same time a strong interaction among stretching, bending, and shearing modes of motion [6]. The dynamic insertion of multiple static equilibria should create phenomena not possible in scalar dynamics supported by the classical Elastica model [1-3] where the inextensibility assumption significantly reduces its predictability. Our results may be useful for understanding dissipation in compliant mechanisms possessing-by design objectives-many equilibrium states [8].

## Predicting Nonlinear Free Dynamics Phenomena with a Physical Model

Figure 1 a photographic view of a physical beam used in experimental studies aimed at predicting interesting nonlinear phenomena in structures with coupled multi-axial dynamics. The physical beam possesses at least seven (7) equilibrium states. On the basis on results for the bifurcation behavior of slow and fast invariant manifolds in coupled linear-nonlinear oscillators-as reduced models of infinite coupled structures [4,5], we sampled the space of vibrations by using state-of-the-art piezoelectric accelerometers and a high performance data acquisition and storage system. *We find that all free coupled vibrations decay very fast*. This fast decay of coupled vector vibrations is accompanied by a continuous, broadband frequency spectrum as discussed in Fig. 2.

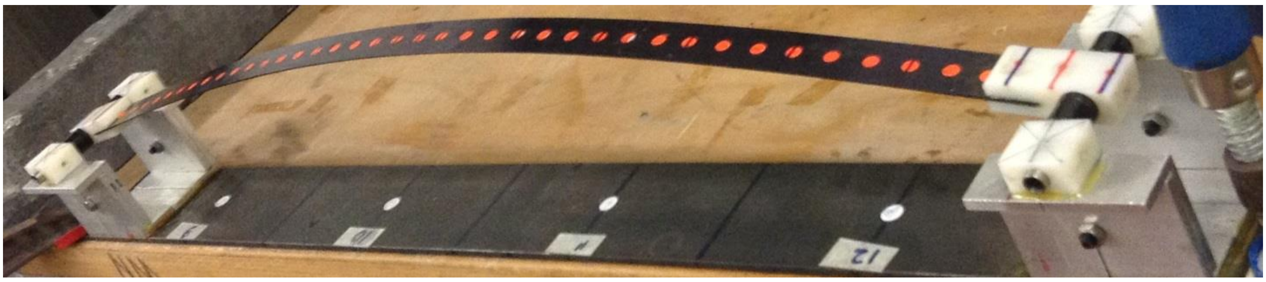

*Figure 1 The elastic beam model (Dynamics-Acoustics & Diagnostics Laboratory, NTUA) used as a physical model to predict nonlinear dynamics phenomena-with interesting energy flow properties-in flexible beams with multiple equilibrium states. The experimental dynamics are three-dimensional with strong nonlinear cross-coupling.*

## Conclusions

We have discovered a remarkable nonlinear dynamics phenomena occurring in a thin elastic beam constrained to move in a plane by imposing a fixed axial distance between its hinged-hinged end points. The discovered dynamics phenomena are free coupled vibrations possessing continuous, broadband frequency spectra. The remarkable characteristic, shared by every motion, is the very fast decay of all free vibrations by attraction to one of the stable equilibrium states. The mechanism of this fast vibration decay-thus energy dissipation-we pinpoint in the interactions of vector vibrations with the multiple equilibrium states. We conjecture that the multiple equilibrium states function dynamically as a generalized scatterer for the initial spatio-temporal configuration of free vibrations.

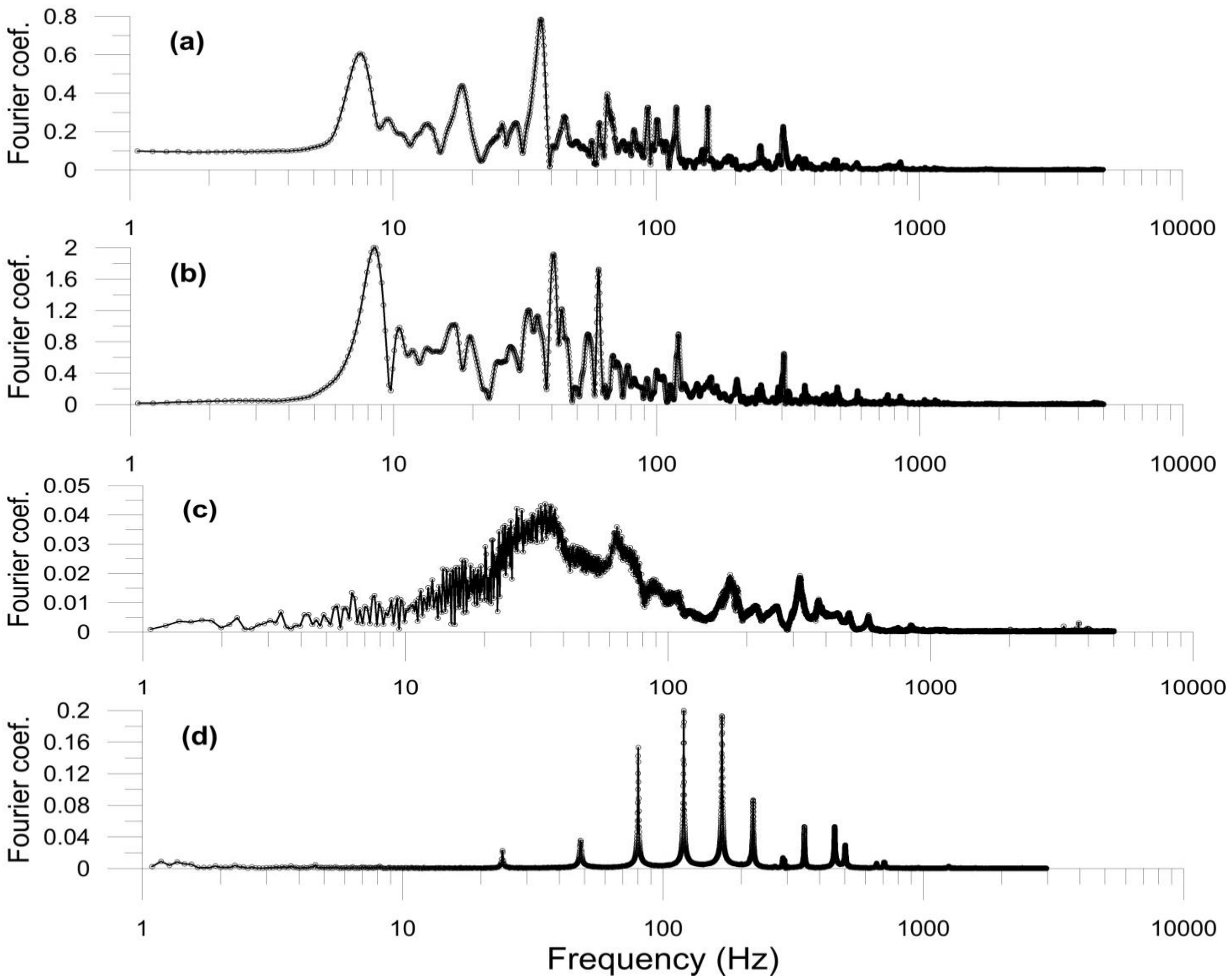


*Figure 2 FFT transforms of measured transverse acceleration during free vector vibrations of the physical beam structure, Fig. 1: (a) local and (b) global vibrations induced by initial displacements; (c) impact-induced global vibration interacting with all equilibria states. This continuous, broadband spectrum is contrasted with the discrete spectrum (d) of a free vibration in a cantilevered straight beam with a single equilibrium state. The discovered continuous, broadband spectra are very similar to those of experimental forced chaotic vibrations of a buckled beam by an axial load reported in [7]. Here the equilibrium states are displacement controlled. This free vibrations phenomenon with ultra-fast decay is reported for the first time here.*

## References


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