Modelling and Control of a Simplified System under External disturbance

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Summary. In various fields of industry, most of the machinery is under the influence of external disturbance such as temperature variation, which affects normal operation of machines. One of the main factors influenced by heat can be thermal expansion or contraction. In this study, a simple traveling non-linear system under the influence of heat from outside is considered and the expansion or contraction of the heat influenced region brings to system malfunction. To make it easy, the system has been simplified and a simple control concept has been applied to this system, to make the system adapt to temperature fluctuation.

System Modelling

Conventional non-linear modelling of roll-to-roll system

A simple roll-to-roll system is considered in this study. Roll-to-roll systems are used in various fields of industry. Example of roll-to-roll system in NPPs are wire rope transferring system in cranes, hoists and cable-reel systems in various electric components. Among many parts of a roll-to-roll system, a single span between two rollers are depicted in Fig. 1. The tension, cross sectional area, density and young's modulus of the traveling part of each span are T, A, ρ, E. Radius, polar moment of inertia and torsional damping of the roller-2 are R, J, c_T. The speed at roller-1 and roller-2 position are v_1 and v_2 respectively. The length of span-2 is L and subject to the influence of dryer. And, in this study it is assumed that the temperature of the traveling part at one instant is the same throughout the span-2.

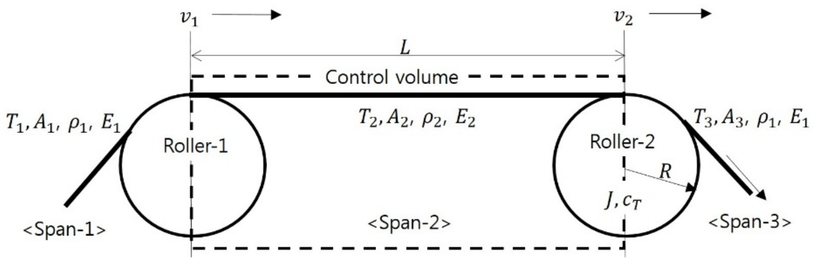

Figure 1: A single span system

From previous studies as Yomick[1], David[2] ,Carlo[3] and Shin[4] a roll-to-roll system can be modeled as Eq.(1). Here, ϵ_i is the strain of the ith web span, M_{mi}, M_{ri} and M_{fi} are motor torque, torque introduced by traveling part and frictional torque at ith roller.

$$\begin{aligned} \frac{d}{dt}\left(\frac{L_i}{1+\epsilon_i}\right) &= -\frac{v_{i+1}}{1+\epsilon_i} + \frac{v_i}{1+\epsilon_{i-1}} \\ \frac{d}{dt}\left(J_i \frac{v_i}{R_i}\right) &= M_{mi} - M_{ri} - M_{fi} \end{aligned} \tag{1}$$

As Yomick[1], David[2] and Carlo[3] mentioned, above system is non-linear. Major factors of non-linearity are elongation due to traveling part weight, friction, width and thickness change of traveling part.

Linearization of non-linearity

The basic concept to get mathematical dynamic model is conservation of mass at the control volume in Fig.1, giving us below equation;

$$\frac{d}{dt}\left(\int_1^2 \rho(x,t)A(x,t)dx\right) = \rho_1(x,t)A_1(x,t)v_1 - \rho_2(x,t)A_2(x,t)v_2 \tag{2}$$

To linearize above model, this study assumed that frictional torque and lateral strain of traveling part are small so that we can ignore them. And by considering real numerical values we can conclude that, strain by traveling part weight is much small than strain by tension and change in thickness is much smaller than longitudinal elongation. So we can ignore them. And if we let the steady state speed at roller-1 and roller-2 position, we can get linearized equation as follows;

$$\dot{T}_2(t) = \frac{EA}{L}V_2(t) \tag{3}$$

However, the corresponding equation by Shin[4] and Hamalainon[5] by considering that the roller-1 rotates in constant speed is as below;

$$\dot{T}_2(t) = \frac{EA}{L}V_2(t) - \frac{v_{20}}{L}T_2(t) \tag{4}$$

By comparing Eq.(3) and Eq.(4) we can notice that there is difference of $\frac{v_{20}}{L}T_2(t)$. And from the free body diagram of roller and traveling part system, we can get equation of motion as Eq.(5).

$$J\dot{V}_2 + c_T V_2 = R^2\big(F(t) - T_2(t)\big) \tag{5}$$

Finally, by substituting Eq.(3) and Eq.(4) into Eq.(5) and by taking Laplace transform, we can get the transfer functions;

$$G(s) = \frac{T_2(s)}{F(s)} = \frac{\beta}{Js^2 + c_T \cdot s + \beta},\ \beta = \frac{EAR^2}{L}\ \text{(method by this study)} \tag{6}$$

$$\mathrm{G(s)} = \frac{\beta}{Js^2 + (c_T + \frac{v_{20}}{L})s + (\beta + \frac{v_{20}}{L})},\ \beta = \frac{EAR^2}{L}\ \text{(method by reference)} \tag{7}$$

Control system and simulation

To conduct simulation, this study used simple PID controller$\left(K = k_p + k_d s + k_i \frac{1}{s}\right)$ and numerical values from reference [4] and [5] are used. Those values are $A = 7.74 \times 10^{-5} m^2$, $E = 2.4GPa$, $J = 10.62Nms^2$, $c_T = 63.7Nms$, $L = 3m$, $R = 10cm$, steady state tension($T_0 = 22.05N$) and steady state speed($v_0 = 0.5\ m/sec$). The heat from outside is considered as temperature disturbance to the system resulting in the tension change having constant thermal expansion coefficient(α) to be $32.3\mu m/(m \cdot ℃)$.
To conduct the simulation, this study used controller specification with overshoot less than 5N and settling time less than 5 seconds. And the temperature disturbance profile used is exponential temperature rise at t=0. Now, to perform the simulation with controller, a trial-and-error approach has been used. As a result, a set of gain with $k_p = 10$, $k_d = 10$, $k_i = 20$ has been chosen. The controlled tension error by using above set of controller gain is shown in Fig. 2. And in this case, c_T=63.7, β=619.35, while $\frac{v_{20}}{L}$=0.1667. So, $c_T + \frac{v_{20}}{L} \cong c_T$ and $\beta + \frac{v_{20}}{L} \cong \beta$ giving Eq.(6)≅Eq.(7). Thus, we can conclude that the method proposed by this study is much simpler to use and it leads almost the same result as conventional method, as shown in Fig.2.

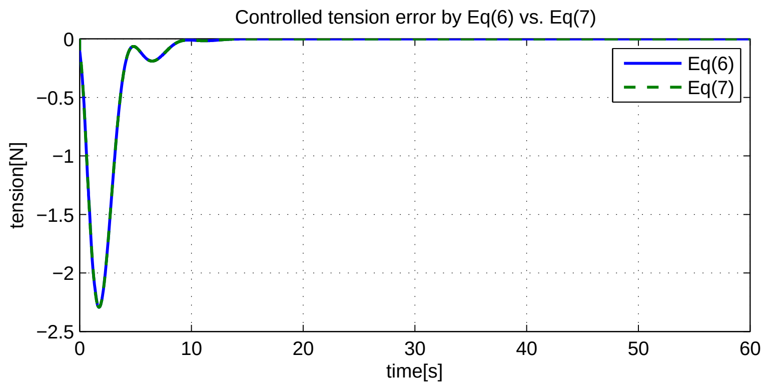

Figure 2: Controlled tension error by temperature disturbance

Conclusions

- A simple model to reflect the effect of the temperature disturbance has been suggested.
- A simple control system to compensate for the effect by temperature disturbance has been studied.
- The model used in this study is simpler and needs less calculation effort than previous method, while giving almost zero deviation to previous method.

References

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