Runout in milling: Tiny cause with significant effects

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Summary. The influence of torsional vibrations on the milling dynamics is studied. In the new model the spindle speed changes in response to the cutting torque. For an ideal process without runout this passive spindle speed variation is periodic with the tooth passing period but does not result in a variable delay, which means that its influence on the milling dynamics is negligible. In contrast, in case of runout, the period of the torsional displacements is equivalent to the spindle rotation period, which results in a variable delay already for the chatter-free cutting solution. A method for the approximation of the resulting time-varying delay is presented, which may significantly change the stability behavior of the milling process dynamics.

Introduction

In milling ideal chatter-free cutting is typically characterized by regular periodic displacements of the structure, where the period is equal to the tooth passing or the spindle rotation period. In contrast, there may occur chatter, which is, typically, characterized by quasi-periodic or chaotic vibrations of the structure with large amplitudes. The prediction of the milling dynamics is important for finding optimal chatter-free cutting conditions by maximizing the material removal rate while avoiding chatter with its detrimental effects such as noise, increased tool wear and bad surface finish. Specifically, the stability lobes in the parameter plane of nominal spindle speed and depth of cut are used to separate cutting conditions corresponding to chatter-free and chattering motion. In applications, typically complex models for the structural dynamics are used for the identification of the stability lobes. On the other hand, many effects in milling dynamics are still unknown. In this contribution, we study the influence of torsional vibrations on the milling dynamics. It turns out that the presence of runout is important for the interpretation of the results and the effects on the stability behavior of the milling process.

Milling process model with torsional flexibility and state-dependent delay

We consider a simple structural model with only one eigenmode for lateral displacements $x(t)$ of the tool (see Fig. 1a). In addition to classical milling models, we take into account torsional displacements $\varphi(t)$ of the tool and study its effects on the cutting process. The influence of torsional vibrations on the stability behavior of the chatter-free tool motion in turning and drilling is studied in [1]. However, in contrast to turning and drilling, the cutting force in milling depends on the angular position $\phi(t) = \Omega_0 t + \varphi(t)$ of the tool, where Ω_0 denotes the nominal spindle speed in rad/s. Specifically, the milling model is given by

$$\ddot{x}(t) + 2\zeta_x\omega_x\dot{x}(t) + \omega_x^2 x(t) = \alpha_x(\Omega_0 t + \varphi(t))F\left(\Omega_0 t + \varphi(t), x(t) - x(t-\theta_\varphi(t))\right), \quad (1)$$

$$\ddot{\varphi}(t) + 2\zeta_\varphi\omega_\varphi\dot{\varphi}(t) + \omega_\varphi^2\varphi(t) = \alpha_\varphi F\left(\Omega_0 t + \varphi(t), x(t) - x(t-\theta_\varphi(t))\right), \quad (2)$$

where ζ_x, ζ_φ and ω_x, ω_φ are the relative damping coefficients and the eigenfrequencies of the dominant eigenmodes in lateral and torsional direction, respectively. The coefficients α_x [kg^{-1}] and α_φ [(m·kg)$^{-1}$] specify the relation between the cutting force F and the excitation of the eigenmodes. In particular, α_φ depends on the moment of inertia and the nominal radius of the tool, and α_x depends on the mass of the tool and the relative orientation between the lateral mode and the cutting force direction. Since the tool is rotating, the coefficient α_x, the chip thickness, and the cutting force depend on the tool rotation angle $\phi(t)$. In addition, the chip thickness and therefore the cutting force depend on the difference between the lateral displacements $x(t)$ at the present cut, and the displacements $x(t-\theta_\varphi(t))$ at the previous tooth pass, which is called regenerative effect (see Fig. 1b). The subscript φ of the time delay $\theta_\varphi(t)$ refers to the state-dependence of the delay. It is defined as the time between the current and the previous cut as [1]

$$\phi(t-\theta_\varphi(t)) = \phi(t) - \Delta \quad \leftrightarrow \quad \theta_\varphi(t) = \theta_0 + \frac{\varphi(t-\theta_\varphi(t)) - \varphi(t)}{\Omega_0}, \quad \theta_0 = \frac{\Delta}{\Omega_0}, \quad (3)$$

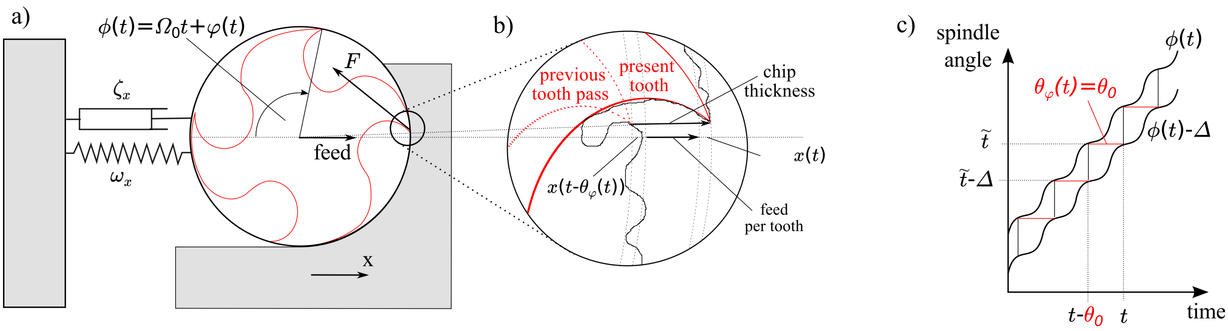

Figure 1: a) Milling process model with lateral $x(t)$ and torsional $\varphi(t)$ displacements. b) Chip thickness with regenerative effect. c) Time delay $\theta_\varphi(t) = \theta_0$ according to Eq. (3) for milling process without runout with the principal period $T = \theta_0$ ($\varphi(t-\theta_0) = \varphi(t)$).

where $\Delta = \frac{2\pi}{Z}$ is the angular distance between two cutting teeth, Z is the number of teeth, and θ_0 is the nominal delay. Eqs. (1)-(3) form a nonlinear system of non-autonomous delay differential equations (DDE) with state-dependent delay. It can be transformed to a DDE with constant delay Δ in the angular domain with the new time scale $\tilde{t} = \phi(t)$ [1].

Effects of torsional vibrations for an ideal tool without runout

For an ideal process without runout the coefficient α_x and the cutting force are periodic with the tooth passing period [2]. In the angular domain we have $\alpha_x(\tilde{t}) = \alpha_x(\tilde{t} + \Delta)$, $F(\tilde{t}, \cdot) = F(\tilde{t} + \Delta, \cdot)$, and the chatter-free solution is a Δ-periodic solution (the period equals the delay). Thus, periodic torsional displacements $\varphi(t) = \varphi(t + \theta_0)$ appear but the corresponding time delay is always constant $\theta_\varphi(t) = \theta_0$; see Eq. (3) and Fig. 1c). In other words, there are passive spindle speed variations but a constant delay equal to the period of the excitation. Indeed, this leads to additional terms in the linearized dynamics compared to the model without torsional vibrations with $\varphi(t) = 0$, e.g. $\alpha_x(\Omega_0 t + \varphi(t))$ appears instead of $\alpha_x(\Omega_0 t)$, and there is the tangential regenerative effect [1], but for generic parameters these modifications does not change the stability lobes significantly (see [1], [3] and Refs. therein).

Effects of torsional vibrations for a tool with runout

In case of runout the period of the forcing is the spindle rotation period [2], which is in the angular domain equivalent to $2\pi = Z\Delta$, i.e. $\alpha_x(\tilde{t}) = \alpha_x(\tilde{t} + 2\pi)$, $F(\tilde{t}, \cdot) = F(\tilde{t} + 2\pi, \cdot)$. In the time domain the period of the system is $T = 2\pi/\Omega_0$ and the delay $\theta_\varphi(t)$ fluctuates periodically around the nominal delay $\theta_0 = T/Z$; cf. Fig. 2a). These passive delay variations are a significant difference to the case without runout or the model without torsional vibrations. The effect on the stability behavior can be analyzed by decoupling the system. Since the regenerative effect is not important for the shape of the torsional displacements $\varphi(t)$, they can be calculated from Eq. (2) by setting $x(t) - x(t - \theta_\varphi(t)) = 0$. The corresponding delay variation $\vartheta_\varphi(t) := \theta_\varphi(t) - \theta_0$ can be determined exactly by Eq. (3) or approximated by Eq. (4); see Fig. 2b),

$$\vartheta_\varphi(t) \approx \frac{\varphi(t - \theta_0) - \varphi(t)}{\Omega_0} \quad \leftrightarrow \quad \frac{\hat{\vartheta}_\varphi(\omega)}{\hat{\varphi}(\omega)} \approx \frac{1}{\Omega_0}\left(\exp\left(-i2\pi\frac{\omega}{Z\Omega_0}\right) - 1\right). \tag{4}$$

After that, the time-varying delay $\theta_\varphi(t)$ can be used in Eq. (1) to calculate the stability lobes with known methods [4]. Typically, the amplitude of the passive delay variation lies between $1\% - 5\%$ of the nominal spindle speed; cf. Fig. 2b). This leads to significant changes of the stability in case of low spindle speeds or large eigenfrequencies of the structure.

Conclusions

A milling model with torsional flexibility was studied, which is described by a system of DDEs with state-dependent delay. For ideal chatter-free cutting without runout no delay variation occurs and the effect of torsional vibrations is negligible. However, in combination with runout a torsional flexibility leads to a periodic delay variation. A suitable approach for the approximation of the variable delay due to runout is proposed, which can be easily extended to complex structural dynamics by using the frequency domain representations of the transfer functions. Then, the effect on the stability lobes can be analyzed with existing methods. It is known that the stability lobes are very sensitive to delay variations, which means that runout may affect the stability especially in the region of low order lobes at low spindle speeds. Recently, we have found that the stability of milling processes with variable pitch and/or variable helix tools is also sensitive to runout [3]. Thus, the effect of runout may be relevant for the stability of milling processes more often than normally expected.

References

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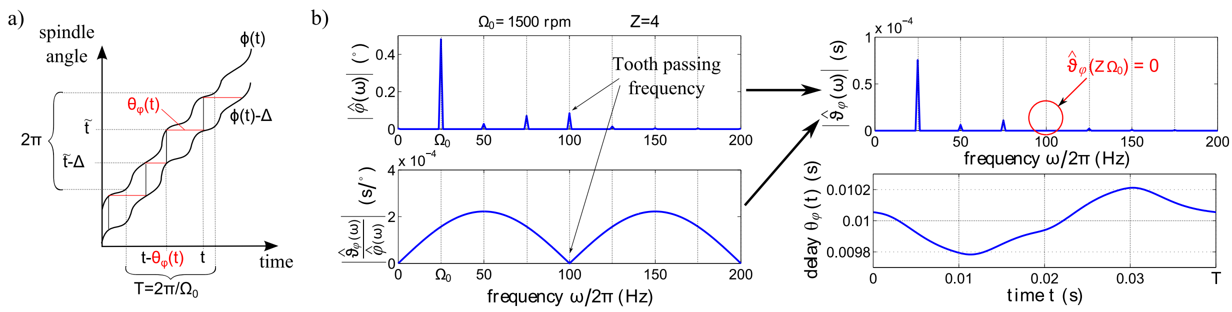

Figure 2: a) Variable delay $\theta_\varphi(t)$ for milling process with runout. b) In case of runout, where the period is the spindle rotation period $T = 2\pi/\Omega_0$, the time-varying delay $\theta_\varphi(t)$ can be determined by Eq. (4) from the torsional displacements $\varphi(t)$.