Bifurcations in implicit map - application to Surface Location Error in milling processes

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Summary. In this contribution, a new type of stability problem is shown in milling operations, which are related to the surface quality. The machined *Surface Location Error* depends on the previously and actually resulted surface offset through the variation of the radial immersion. Bifurcation analysis is presented for the governing nonlinear implicit map.

Introduction

In industry, milling is a widely used manufacturing method. Due to the intermittent behaviour of the milling processes, the cutting edges enter and exit the material periodically according to the applied radial immersion. This phenomenon leads to a periodic cutting force which always results forced vibration in the mechanical system. This vibration is copied to the surface and creates the so-called *Surface Location Error* (*SLE*), which is an offset error defined by the largest deviation between the machined and the desired surface [1, 2]. Since the forced vibration usually creates negligible surface roughness, this error could be compensated with a proper tool path modification. This type of surface error is significant typically at resonant spindle speeds where large amplitude vibration can occur. During the prediction of this surface error, the widely used methods do not consider that the *SLE* can influence the radial immersion. However, in the case study [1], the measured vibration amplitude was in the range of the radial immersion. Moreover in case of consecutive immersions during roughing process, the actual and the preceding *SLE* modifies the entering and the exiting position. In a previous study [3], the evaluation of the series of *SLE* values is investigated in a way that the actual pre-set radial immersion is modified by the *SLE* of the previous immersion only, which leads to an explicit map. In this paper, the actual *SLE* is considered by means of an implicit map, and the stability properties of the fix point and corresponding bifurcation curves are analysed.

Implicit map

The *SLE* depends on various dynamical and cutting parameters like feed per tooth, spindle speed, axial depth of cut, radial immersion and milling type such as up- or down-milling. The radial immersion has a key role in the following analysis, therefore, a function is defined which describes the dependence of the *SLE* on the radial immersion. It is denoted by $f_{\mathrm{SLE}}(a) = \mathit{SLE}$, where a is the radial immersion (see Fig. 1b). The derivation of the *SLE* for a given radial immersion is not described in this paper; several methods can be found in the literature [1, 2]. The *Surface Location Error* for the i^{th} consecutive immersion can be calculated by a map as $\mathit{SLE}_i = f_{\mathrm{SLE}}(a_i)$, where the current radial immersion a_i is composed by the pre-set radial immersion a_0, the actual error SLE_i and the previously resulted error SLE_{i-1} in the form as $a_i = a_0 + \mathit{SLE}_{i-1} - \mathit{SLE}_i$ (see Fig. 1a). Note, there is an analogy between this model and the surface regenerative effect [4], where the actual chip thickness depends on the feed, the present and the delayed positions of the tool. To compare the phenomena, it can be modelled by delay differential equations, but the evaluation of the *SLE* can be given as a difference equation by means of an implicit map, given in the form,

$$\mathit{SLE}_i = f_{\mathrm{SLE}}(a_0 + \mathit{SLE}_{i-1} - \mathit{SLE}_i). \tag{1}$$

This implicit map determines how an *SLE* develops into another *SLE* over immersion-by immersion. These series of the *SLE*s may converge to a fix point, called *Cumulative Surface Location Error* ($\mathit{CSLE}_{\mathrm{p1}} = \lim_{i\to\infty} \mathit{SLE}_i$), where p1 stands for the period-1 or fix point solution. At these fix points, the following expression holds; $\mathit{SLE}_i = \mathit{SLE}_{i-1} \equiv \mathit{CSLE}_{\mathrm{p1}}$, therefore the fix point $\mathit{CSLE}_{\mathrm{p1}}$ is the given function $f_{\mathrm{SLE}}(a_0) = \mathit{CSLE}_{\mathrm{p1}}$ (see Fig. 1b). The stability of this fix point is determined by perturbation method in the next Section.

Figure 1: a) schematic model of the surface evaluation in successive immersions; b) resulted surface error in the function of the dimensionless radial immersion

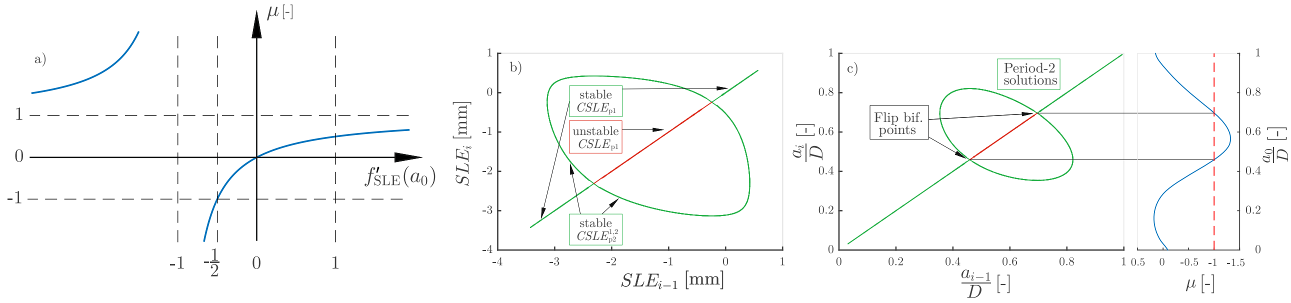

Figure 2: a) decay ratio of the perturbation of the fix point; b),c) stable, unstable fix points and period-2 solutions respected to the surface errors and to the radial immersion together with the decay ratio of the corresponding fix point

Stability analysis

The general discrete solution SLE_i can be written as a small perturbation ξ_i around fix point as $SLE_i = CSLE_{\mathrm{p1}} + \xi_i$. Substituting into Eq. (1), expanding into Taylor series and eliminating the higher-order terms give the implicit variational map. However, this implicit map is linear, therefore it can be transformed into explicit formula, read as:

$$\text{implicit formula: } \xi_i = f'_{\mathrm{SLE}}(a_0)(\xi_{i-1} - \xi_i), \quad \text{explicit formula: } \xi_i = \underbrace{\frac{f'_{\mathrm{SLE}}(a_0)}{f'_{\mathrm{SLE}}(a_0)+1}}_{=\mu} \xi_{i-1} \tag{2}$$

In the explicit formula, μ denotes the decay ratio. If $|\mu| < 1$ then the fix point is stable, otherwise it is unstable [5]. As it is shown in Eq. (2), μ can be expressed as a hyperbolic function of the derivatives of the $f_{\mathrm{SLE}}(a_0)$ at the fix point (see Fig. 2a). As it can be seen, that flip bifurcation can occur at $f'_{\mathrm{SLE}}(a_0) = -0.5$, which leads to period-2 solutions around the fix point, for which $CSLE^{1,2}_{\mathrm{p2}}$ denotes the two alternating solutions. Note, that usual fold bifurcation cannot occur since it is not possible that μ reach 1. To obtain period-2 solutions, two-step map have to be prepared, therefore Eq. (1) is substituted successively into itself, reads as

$$SLE_{i+2} = f_{\mathrm{SLE}}(a_0 + \underbrace{f_{\mathrm{SLE}}(a_0 + SLE_i - SLE_{i+1})}_{SLE_{i+1}} - SLE_{i+2}) \tag{3}$$

In case of period-2 solutions, the following expressions hold: $SLE_i = SLE_{i+2} \neq SLE_{i+1} \equiv CSLE^1_{\mathrm{p2}}$ and $SLE_{i+1} = SLE_{i-1} \neq SLE_i \equiv CSLE^2_{\mathrm{p2}}$. Substituting into Eq. (3), period-2 solutions $CSLE^{1,2}_{\mathrm{p2}}$ can be calculated by means of solving the following equations

$$\begin{aligned} CSLE^1_{\mathrm{p2}} &= f_{\mathrm{SLE}}(a_0 + f_{\mathrm{SLE}}(a_0 + CSLE^1_{\mathrm{p2}} - CSLE^2_{\mathrm{p2}}) - CSLE^1_{\mathrm{p2}}) \\ CSLE^2_{\mathrm{p2}} &= f_{\mathrm{SLE}}(a_0 + f_{\mathrm{SLE}}(a_0 + CSLE^2_{\mathrm{p2}} - CSLE^1_{\mathrm{p2}}) - CSLE^2_{\mathrm{p2}}). \end{aligned} \tag{4}$$

Note, that the stability of these period-2 solutions can be analysed in the same way as presented above. If further bifurcations are detected on a period-2 branch, then period-4 branch will occur, and the similar procedure have to be applied with four-step map. The above-described computation method is applied to a case study and presented in Fig. 2. The calculated stable and unstable fix points $CSLE_{\mathrm{p1}}$ are shown in Fig. 2bc along the 45° line with green and red color, respectively. The stable period-2 solutions $CSLE^{1,2}_{\mathrm{p2}}$ are also denoted. The decay ratio μ of the fix point is visualised in Fig. 2c, which shows that the stable fix points become unstable and create period-2 solutions at $\mu = -1$.

With the proposed model it is shown that the series of *SLE* can be a period-2 solution, which leads to an unpredictable surface error at the end of the roughing processes.

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