Synchronisation of beams attached to a rotating hub

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Summary. The aim of the paper is to analyse the synchronisation phenomenon of a rotating structure comprising three composite beams attached to a rigid hub. In the analysis authors have assumed that one rotor beam is 10% thicker comparing to the remaining ones. Furthermore, non-classical effects as material anisotropy, transverse shear and cross-section warping are taken into account. The partial differential equations of motion (PDEs) of the system are derived by Hamilton's principle and then reduced to ordinary differential equations (ODEs) by the Galerkin's method. Next, these are solved numerically and resonance curves for individual beams and hub are plotted.

Introduction

Rotating structures are very well known in mechanical engineering, typical examples are wind turbines, helicopter rotors and jet engines. Dynamics of a thin-walled composite rotating structure in linear regime has been studied in [2, 4]. Nonlinear dynamics and synchronisation effect observed in rotating structures composed of two pendulums attached to a hub has been discussed in [5]. The famous Huygens experiment with synchronisation of two clocks have been recalled in [3], where the basic definition of synchronisation phenomenon has been given. Moreover, synchronisation of the pendulums rotating in concurrent directions has been presented in [1].

Model and governing equations of the structure

The paper considers a system comprising three elastic composite beams attached to the rigid hub as shown in the Figure 1. The hub of radius R_0 may rotate or oscillate about vertical axis OZ_0, the current position of the hub is given by an angle ψ. Each beam is described by its length, the cross-section width and thickness as l_i, d_i and h_i respectively, where $i = 1, 2, 3$. The Hamilton's principle was used to derive the system of partial differential equations for each beams and for

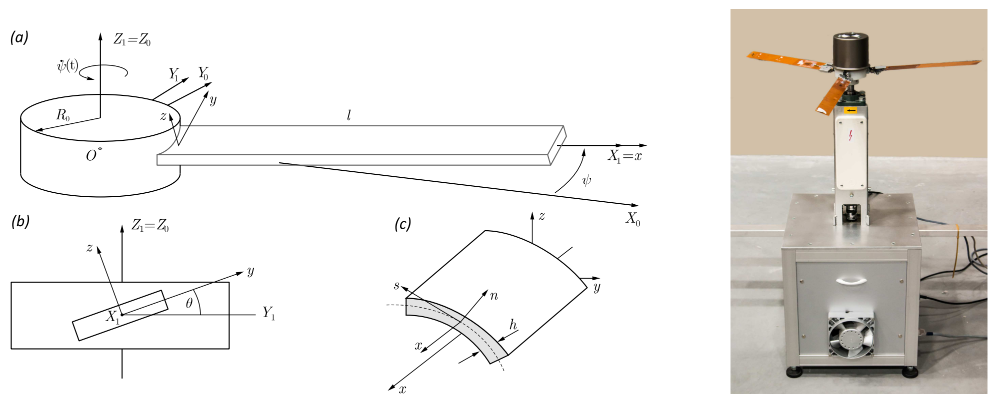

Figure 1: Model of the individual elastic composite beam and experimental setup for 3 blades rotor testing

the hub. Reduction of PDEs to the ordinary differential equations (ODEs) was done by the Galerkin projection method yields:

$$
\begin{aligned}
&(1 + J_h + J_{b2} + J_{b3} - \alpha_{h12}q_1 - \alpha_{h22}q_2 - \alpha_{h32}q_3)\dot{\Omega} + \alpha_{h11}\ddot{q}_1 \\
&+\alpha_{h21}\ddot{q}_2 + \alpha_{h31}\ddot{q}_3 - \alpha_{h13}\dot{q}_1\Omega - \alpha_{h23}\dot{q}_2\Omega - \alpha_{h33}\dot{q}_3\Omega + \zeta_h\Omega = \mu \\
&\ddot{q}_1 + \alpha_{12}\dot{\Omega} - \alpha_{14}\dot{q}_1\Omega + (\alpha_{11} + \alpha_{13}\Omega^2)q_1 + \zeta_1\dot{q}_1 = 0 \\
&\ddot{q}_2 + \alpha_{22}\dot{\Omega} - \alpha_{24}\dot{q}_2\Omega + (\alpha_{21} + \alpha_{23}\Omega^2)q_2 + \zeta_2\dot{q}_2 = 0 \\
&\ddot{q}_3 + \alpha_{32}\dot{\Omega} - \alpha_{34}\dot{q}_3\Omega + (\alpha_{31} + \alpha_{33}\Omega^2)q_3 + \zeta_3\dot{q}_3 = 0
\end{aligned} \tag{1}
$$

The first equation of the system (Eqs. 1) describes the dynamics of the hub. The J_h denotes a ratio of the hub inertia to the inertia of the first beam. Similarly J_{b2} and J_{b3} are relative inertia of the second and the third beam with respect to the first one. Ω represents the dimensionless angular velocity of the hub. The system is excited by driving torque μ expressed as $\mu = \mu_0 + \rho \sin \omega t$. Last three equations describe the beams dynamics, where q_1, q_2 and q_3 are generalized coordinates corresponding to the complex coupled flexural–torsional specimen deformation. Damping of the system is included by arbitrary introduced viscous damping coefficients ζ_1, ζ_2, ζ_3 and ζ_h for beams and the hub, respectively.

Numerical studies

Numerical studies have been performed for the three beams rotor system, considering two identical beams (No.1 and 3) and one de-tuned, due to 10% higher thickness (No.3). All α_{ij} factors present in equations of motion (Eqs. 1) have been calculated for physical model of the rotating structure.

Table 1: Dimensionless coefficient, based on the real physical model

$\alpha_{11} = \alpha_{31}$	12.0388808306	α_{21}	14.5664355288	$\alpha_{h11} = \alpha_{h31}$	0.0112028592	α_{h21}	0.0123216629
$\alpha_{12} = \alpha_{32}$	1.9765664564	α_{22}	1.9768127528	$\alpha_{h12} = \alpha_{h32}$	0.0084706516	α_{h22}	0.0093152417
$\alpha_{13} = \alpha_{33}$	0.4946581624	α_{23}	0.4946272964	$\alpha_{h13} = \alpha_{h33}$	0.0084706516	α_{h23}	0.0093152417
$\alpha_{14} = \alpha_{34}$	0.7758539801	α_{24}	0.7757419119	ζ_h	0.1	ζ_i	$0.001\sqrt{\alpha_{i1}}$

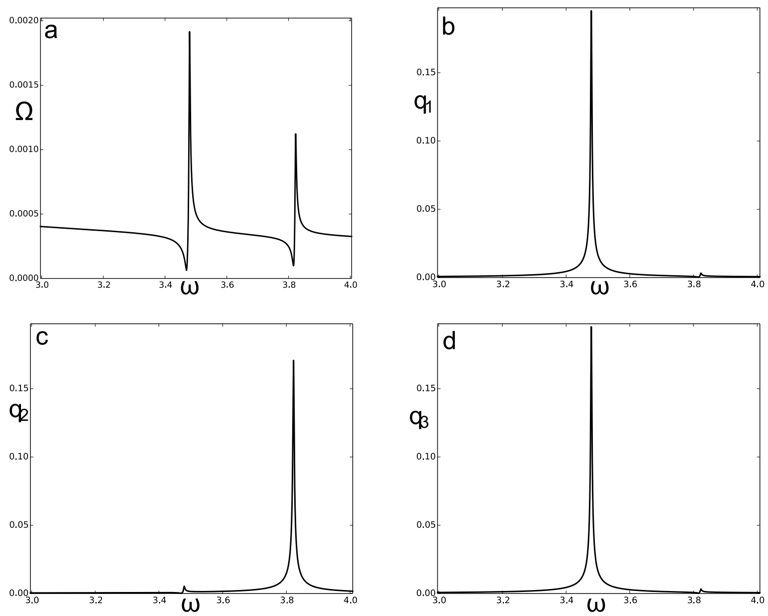

Figure 2: Resonance curve for the amplitude of excitation $\rho = 0.001$; a) angular velocity of the hub Ω, b) displacement of the first beam q_1, c) second beam q_2 and d) third beam q_3

Initial numerical simulations have been performed for the arbitrary selected excitation amplitude $\rho = 0.001$. Hub response is shown in the Figure 2a, with two resonances, first close to $\omega \approx 3.5$ and second $\omega \approx 3.8$. Resonance curves for individual beams are presented in Figures 2b,c,d by amplitude of coordinate q_i. The resonances occurred for the same value of ω like for the amplitude of angular velocity of hub. Response of the second beam q_2, due to higher thickness is different from the other two beams.

Conclusions

Studying given above results one observes two resonances. The first one is close to $\omega = 3.5$ and the second occurs approx. at $\omega = 3.8$. Resonance curves for amplitudes of the first and the third beam are exactly the same (q_1, q_3) due to the symmetry. However, the main resonance for the beam number two is the second one, which is different from the other beams. At the first resonance beams number one and three exhibit the complete synchronisation, while for the second beam synchronisation with locked phase is observed. In case of the second resonance area similar scenario is evident.

Acknowledgment

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