Control of a vertical mode of a cable by a nonsmooth oscillator

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Summary. The aim of this work is controlling vertical modes of a cable via a nonsmooth (piece-wise linear) oscillator. Continuous system equations are discretized via projections of the equations in physical domain on modal domain. A time multiple scale method (not presented here) has been used to trace system behaviors at different scale: Obtained slow invariant manifold of the system at fast time scale provides a good insight for tuning of such nonsmooth systems to able to activate them rapidly. Detected characteristic points of the system around the invariant prepare necessary design tools for having desirable responses or regimes (periodic or modulated regimes) during extreme nonlinear energy exchanges between two coupled oscillators. The paper finally demonstrates experimental results of the control process of first vertical mode of a cable via designed nonsmooth oscillator by mentioned techniques.

Mathematical model of the system and projection of the system on the targeted mode

Let us consider a cable structure which is illustrated in Fig. 1. Governing system equations in the absence of external excitations are summarized as (H is the constant horizontal component of the tension of the cable and m_l is the linear density):

$$H\frac{\partial^2 \upsilon(X,t)}{\partial X^2} = m_l \frac{\partial^2 \upsilon(X,t)}{\partial t^2} \tag{1}$$

The system is composed of infinite modes, with frequencies as $\omega_{|k}$ and modal deformations as $X \to \upsilon_{|k}(X)$, $k \in \mathbb{N}$. We suppose that the particular solution of the Eq. 1 reads as: $(X,t) \to \upsilon_{|k}(X)e^{i\omega_{|k}t}$. Let us set: $\upsilon(X,t) = \sum_{k=1}^{\infty} q_{|k}(t)\upsilon_{|k}(X)$ and inject it in the Eq. 1; we will have:

$$\frac{\partial^2 q_{|k}}{\partial t^2} + \omega_{|k}^2 q_{|k} = 0 \tag{2}$$

If the system is under external forcing $E(t)$ at the point $X_{for}(X)$, then governing system equations will take following form ($\delta_{X_{for}}$ stands for the Dirac function):

$$H\frac{\partial^2 \upsilon}{\partial X^2} + E(t)\delta_{X_{for}}(X) = m_l \frac{\partial^2 \upsilon}{\partial t^2} \tag{3}$$

If one follows explained above mentioned procedure for treating the case of free vibration, following system can be obtained for the forced system:

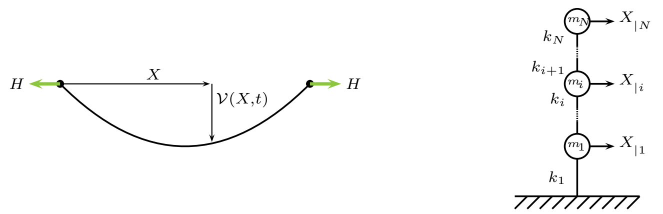
$$m_l < \upsilon_{|k}|\upsilon_{|k} > \frac{\partial^2 q_{|k}}{\partial t^2} + m_l < \upsilon_{|k}|\upsilon_{|k} > \omega_{|k}^2 q_{|k} = E(t) < \delta_{X_{for}}|\upsilon_{|k} > \tag{4}$$

Let us define: $\bar{X} = q_{|k}X_{car}$, $M = \dfrac{m_l < \upsilon_{|k}|\upsilon_{|k} >}{X_{car} < \delta_{X_{for}}|\upsilon_{|k} >}$, $K = M\omega_{|k}^2$. Equation 4 yields to:

$$M\frac{d^2\bar{X}}{dt^2} + K\bar{X} = E(t) \tag{5}$$

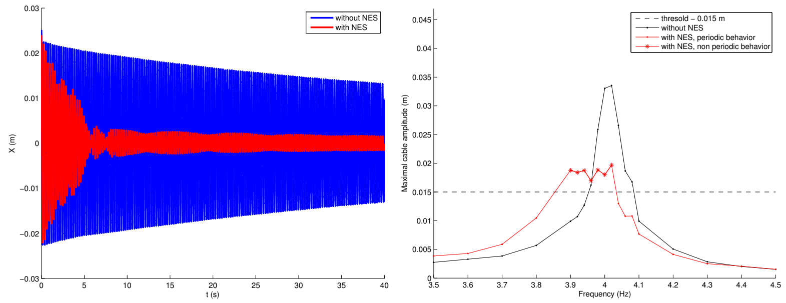
If we consider a damping for the system (i.e. $\Lambda = 2\omega_{|k}M\xi_{|k}$), then we will have:

$$M\frac{d^2\bar{X}}{dt^2} + \Lambda\frac{d\bar{X}}{dt} + K\bar{X} = E(t) \tag{6}$$



(a) The cable structure (b) Discretized model of the cable

Figure 1: The continues model of the cable and its discretized model in the form of projection on its modes.



(a) Free vibration

(b) Forced vibration

Figure 2: The control process of the first vertical mode of a cable by a designed nonsmooth NES under free and forced vibrations. For the forced system with NES, different possible regimes, periodic or modulated responses, can be faced.

We define a new time domain as $\tau = \omega_{|k}t$ and new function for displacement as $\bar{X} = X_{ad}x$. Moreover, we suppose that $\epsilon\lambda = 2\xi_{|k}$ and $E(t)$ is sinusoidal, then we will have:

$$\ddot{x} + \epsilon\lambda\dot{x} + x = \epsilon f \sin(\omega\tau) \tag{7}$$

We couple a nonlinear energy sink (NES) [1] with a general restoring forcing function as C_1F applied at the point $\delta_{X_{NES}}(X)$ $(X_{car} = \upsilon_{|k}(NES) \Rightarrow \bar{X} = q_{|k}\upsilon_{|k}(NES))$:

$$\begin{cases} H\dfrac{d^2\upsilon}{dX^2} + E(t)\delta_{X_{for}}(X) + C_1F(\upsilon(X_{NES}, t) - Y(t), ...)\delta_{X_{NES}}(X) = m_l\dfrac{d^2\upsilon}{dt^2} \\ \dfrac{d^2Y}{dt^2} - C_1F(\upsilon(X_{NES}, t) - Y(t), ...) = 0 \end{cases} \tag{8}$$

If damping scenarios for both systems are supposed and considering above mentioned change of variables, Eq. 8 reads:

$$\begin{cases} M\dfrac{d^2\bar{X}}{dt^2} + 2M\omega_{|k}\xi\dfrac{d\bar{X}}{dt} + K\bar{X} + C_1F(X - Y, ...) + 2M\omega_{|k}\xi_1(\dfrac{d\bar{X}}{dt} - \dfrac{dY}{dt}) = E(t)\dfrac{< \delta_{X_{for}}(X)|\upsilon_{|k} >}{< \delta_{X_{NES}}(X)|\upsilon_{|k} >} \\ \dfrac{d^2Y}{dt^2} - C_1F(X - Y, ...) - 2M\omega_{|k}\xi_1(\dfrac{d\bar{X}}{dt} - \dfrac{dY}{dt}) = 0 \end{cases} \tag{9}$$

which provides (p_m is the effect of the gravity which can not be ignored since we consider a vertical system):

$$\begin{cases} \ddot{x} + \epsilon\lambda\dot{x} + x + \epsilon\lambda_1(\dot{x} - \dot{y}) + \epsilon c_1F(x - y, ...) + p_m = \epsilon f \sin(\omega\tau) \\ \epsilon\ddot{y} + \epsilon\lambda_1(\dot{y} - \dot{x}) + \epsilon c_1F(y - x, ...) + \epsilon p_m = 0 \end{cases} \tag{10}$$

Experimental results

Validity of the modeling has been tested so that the two degrees of freedom system (1 mode + 1 NES) could be considered for the design. A *nonsmooth* (piece-wise linear) vertical NES in term of its restoring function, i.e. $c_1F(z, ...)$, has been designed and fabricated. The design was based on time-multiple scale detection of system responses [2] and finding its equilibrium and singular points which correspond to periodic and modulated responses [3]. Weiss et al. [4] presented similar design procedure for controlling horizontal modes of cables by NES with *cubic* nonlinearity without considering the effects of the gravity. Here, the goal was to control first vertical mode of the cable in the presence of the gravity with nonsmooth NES. Obtained experimental results are given in Fig. 2 showing that the designed nonsmooth NES is able to control first vertical mode of the cable leading to final periodic regimes or modulated responses.

Conclusions

A nonsmooth nonlinear energy sink has been designed and fabricated for controlling the first vertical mode of a cable. The design procedure contains time multi-scale detection of system responses: fast time scale provides slow invariant manifold while slow time scale traces all characteristic points of the system, i.e. equilibrium and singular points. The designed nonsmooth NES controlled the cable against imposed initial conditions and also under external forcing terms.

References

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