Delay-Independent Partial Synchronization in Networks of Non-identical Nonlinear Systems with Transmission Delay Coupling

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Summary. This paper considers delay-independent synchronization in networks of nonlinear systems with communication delay couplings. We have already shown that depending on the parity of the number of systems, either partial synchronization or full synchronization occurs in ring networks of identical systems regardless of the length of time-delay. In this paper, using a similar approach, we show that if the network is isomorphic to a biregular bipartite graph, delay-independent synchronization occurs in the network systems. Since bipartite graphs are equivalent to two-colorable graphs, and the pattern of partial synchronization is the same as the color coding of the graph. Also, even if different dynamics is allocated to the different colored nodes, the partial synchronization appears regardless of the delay. The validity of the obtained results is supported by numerical examples.

Extended Abstract

Recently, synchronization in networks of dynamical systems has attracted a great deal of attention in various fields. The synchronization problem can be solved by considering the following points: (1) the boundedness of the solution of coupled systems, (2) the existence of the synchronization manifold, and (3) the stability of the synchronization manifold. Therefore the derivation of synchronization condition in networks has linked the stability analysis to the underlying structural properties of networks. Since the existence of time-delays in systems affects the stability of systems in practical situations, the synchronization condition for coupled systems with delay couplings depends on not only the coupling strength but the length of delay in most cases. However, if the delayed coupling is a transmission delay coupling defined by

$$u_i(t) = -k\sum_j (y_i(t) - y_j(t-\ell)),$$

synchronization may occur regardless of the length of time-delay under a certain condition.
In our recent works, we focus on delay-independent synchronization that may occur in networks of systems with transmission delay couplings. The possibility of such synchronization in networks of homogeneous nonlinear systems with transmission delay couplings was pointed out in several works [1],[2],[5], [6],[7]. In [5], we derived a sufficient condition such that the synchronization error dynamics does not explicitly have time-delay. However, even if the synchronization error dynamics has time-delay explicitly, the delay-independent synchronization can occur depending on the network structure. In [6], we considered the synchronization problem for semi-passive systems with relative degree one in ring networks of bidirectional transmission delay couplings. For such systems, we showed that if the number of systems is odd, then full synchronization occurs regardless of the length of time-delay for a coupling strength larger than a certain value. Also, we showed that delay-independent partial synchronization can be observed in ring networks of an even number of systems.
In this paper, first we consider N identical systems coupled with transmission delay couplings. Then, we show that if the network structure is represented by a biregular bipartite graph, that is a bipartite graph in which all vertices on the same side of the bipartition have the same degree, delay-independent synchronization occurs among all systems on each side of bipartition. For instance, a network shown in Fig. 1 is isomorphic to the corresponding biregular bipartite graph as shown in Fig. 2. Then, all nodes are classified into two sets, and all systems in each set synchronize regardless of the length of time-delay. As is known well, bipartite graphs are equivalent to two-colorable graphs. Therefore, if the network is two-colorable and all the same-colored nodes have the same degree, the same-colored nodes synchronized independently of the length of time-delay.

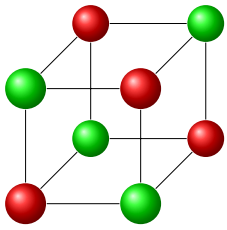

Figure 1: 8-node network ($N = 8$)

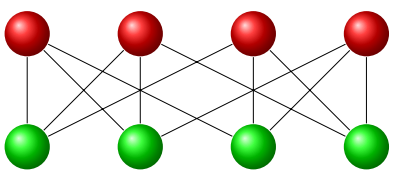

Figure 2: The corresponding bipartite graph

Next, we consider networks of non-identical systems with transmission delay couplings. Then, we show that if all the same-colored nodes on two-colorable graphs have the same degree and they have the same dynamics, delay-independent partial synchronization occurs in the networks. The synchronization pattern is the same as the color coding of the graph. As a numerical example, we consider a synchronization condition for the network system shown in Fig. 1. The FitzHugh-Nagumo (FHN) model is allocated on each red-colored node in this figure, and the Hindmarsh-Rose (HR) model is on each green-colored node.
Figure 3 shows a synchronization condition obtained through numerical simulations. In this figure, the yellow region indicates that all the systems with the same dynamics synchronize, but FHN systems and HR systems do not synchronize. As a result, we can observe delay-independent partial synchronization with the pattern as shown in Fig. 1.

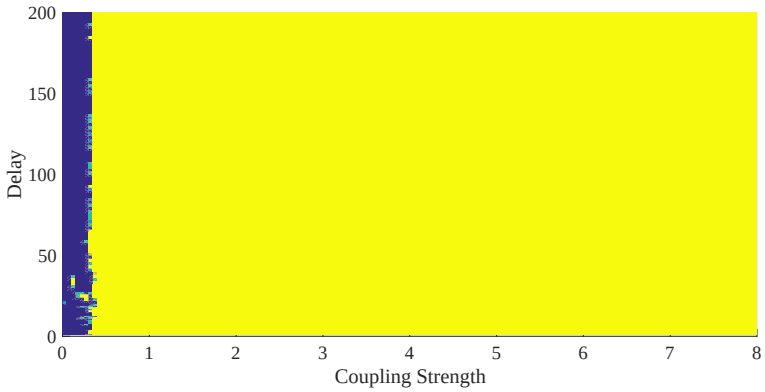

Figure 3: Synchronization region

This result supports the validity of our obtained results through numerical simulations.

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