

Nonlinear Dynamics of Oscillators with Shape Memory Alloy

Sebastian Tatzko, Jonas Böttcher

Institute of Dynamics and Vibration Research, Leibniz Universität Hannover, Hannover, Germany

Summary. The stress-strain-curve of superelastic shape memory alloys (SMA) exhibits a hysteresis which offers the possibility to increase structural damping with SMA's. This paper shows the strongly nonlinear behavior of a single degree of freedom oscillator with SMA.

Introduction

The hysteresis loop of the stress-strain-curve of shape memory alloys (SMA) offers the possibility to exploit SMA material for vibration damping. In Figure 1 a) a measured stress-strain-curve of an SMA wire is shown with a piece-wise linear approximation [1]. In [2] a method is described that allows to calculate the possible damping performance of SMA's based on the piece-wise linear hysteresis. The fundamental dynamics of vibration systems with SMA can be analyzed by a single degree of freedom (SDOF) oscillator where the linear spring is replaced by a nonlinear force element capturing the SMA hysteresis, see Figure 1 c), [3], [4]. An overview on how shape memory alloys can be employed for dynamical applications is given in [3]. In [1] the harmonic balance method (HBM) is applied to the SDOF system and FRF's shown in Figure 1 b) were obtained which reveal the strongly nonlinear forced excitation response. Increasing the excitation level the system shows at first a softening behavior and at a certain point a hardening behavior. In the latter region the system can exhibit the jump phenomenon. For more information on the characteristics of SMA oscillators with various types of bifurcations explained see [4].

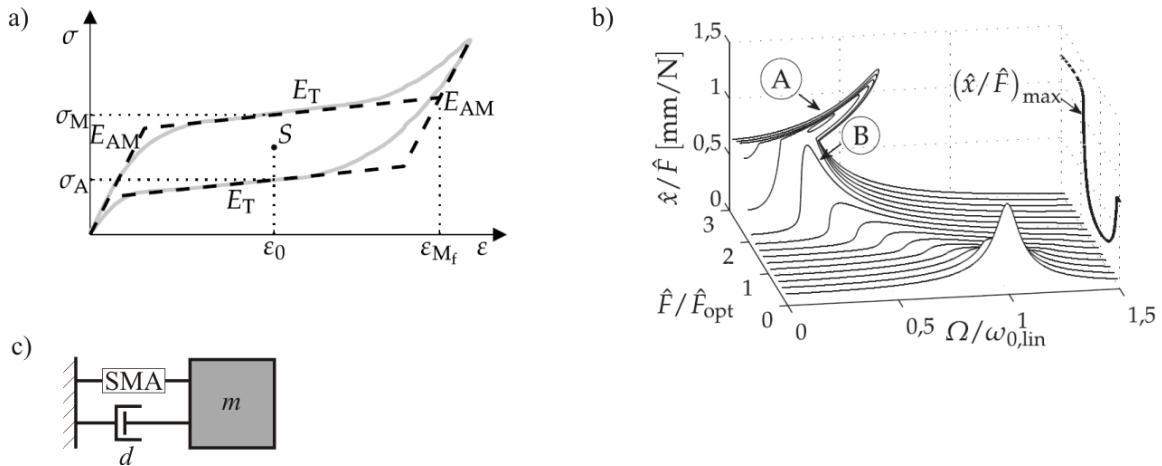


Figure 1: a) Stress-strain-curve of a SMA wire with piece-wise linear approximation from [1], b) FRF's of SDOF oscillator with SMA from [1], c) SDOF oscillator with SMA

Modeling and Parameters

In this work the nonlinear behavior of the oscillator from Figure 1 c) is analyzed by means of frequency sweeps. The system shown in Figure 1 c) is used with a piecewise linear hysteresis loop, see Figure 1 a). A model of the nonlinear oscillator with forced excitation is set up in MATLAB SIMULINK which allows for time integration for given initial conditions. The parameters are chosen as follows. The oscillator mass is $m = 1$ kg and the linear damping value $d = 0.1$ Ns/m. The SMA hysteresis is modeled as a nonlinear restoring force where the stiffness corresponding to E_T in Figure 1 a) is zero and the stiffness corresponding to E_{AM} is 1 N/m. The Force value corresponding to $\sigma_M - \sigma_A$ is chosen to be 0.2 N. With this set of parameters the characteristics of the nonlinear oscillator are investigated.

Simulation results

Frequency sweeps are conducted for different levels of the forcing amplitude and the results are summed up in Figure 2. The calculated responses are each divided by the excitation force amplitude, see also Figure 1 b). The softening-hardening behavior is clearly visible and the jump phenomenon is captured by forward and backward sweeps in the region with high level excitation. The calculated hysteresis curves of the forward sweeps are shown for the three characteristic regions of the nonlinear oscillator. The low level excitation corresponds to a purely linear system with linear FRF. In the medium level excitation region the SMA behaves like a friction damper (Coulomb slider model with elasticity) and at high level excitation the hysteresis shape can be compared to compliant unilateral contact.

A closer look at the high level excitation shows that after the jump the position of rest of the system is ambiguous. There

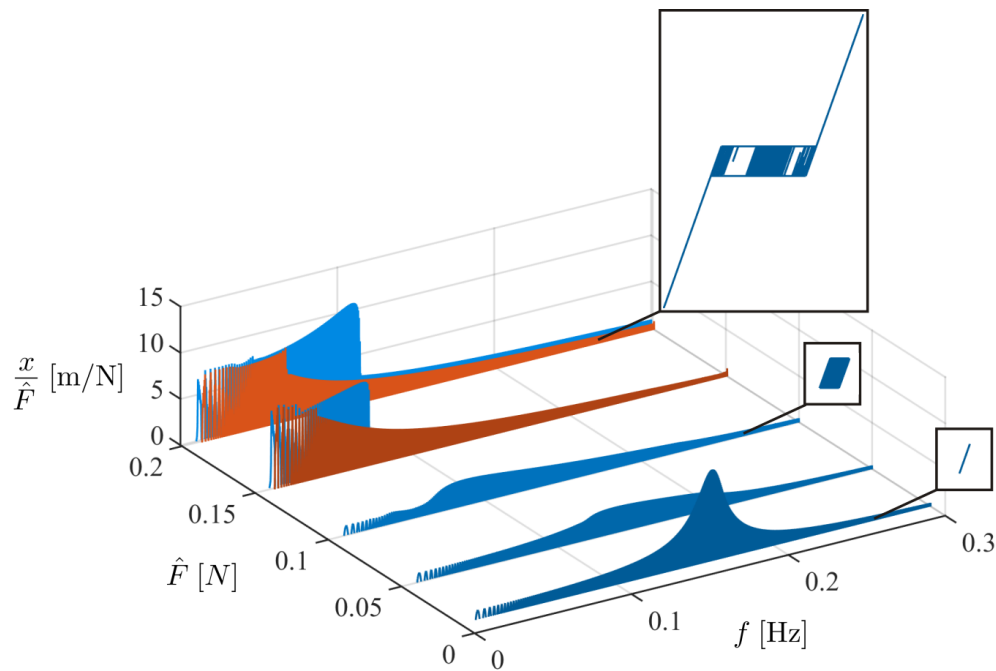


Figure 2: Time integration results

is a shift between forward and backward sweep and the path in the corresponding hysteresis of the forward sweep is consequently not symmetric.

Conclusions

Oscillators with shape memory alloy show a strongly nonlinear behavior. Simulation results obtained by time integration in MATLAB SIMULINK are in good agreement with the characteristics of SMA oscillators found in [1]. The results further show that the harmonic balance approximation used in [1] considering the fundamental harmonic only is able to capture the characteristic softening-hardening behavior of the frequency response.

References

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