Power Generation of a Pendulum Energy Converter excited by Random Loads

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Summary. We present a case study on the energy generation of a novel energy harvester design based on a pendulum whose pivot is randomly excited. Thereby, we deal with a random excitation by a non-white Gaussian stochastic process. We analyze different experimental setups in order to maximize the harvested energy of the pendulum energy converter.

Introduction

A fundamental problem in the field of renewable energy is energy harvesting from a source, whose nature is random. Since wave energy has been recognized as one of the most promising resources of renewable energy due to the notably high power density of 9.42 kW/m^2 compared with wind and solar energy with power density of 0.58 and 0.17 kW/m^2, respectively [1], several new concepts of ocean wave energy converters were recently developed and studied, for example [2, 3]. One possible method of energy generation is to use ocean gravity waves to excite the pivot of a pendulum in order to induce and maintain rotational motion by means of a control method, as described in [2, 3]. The induced motion can then be converted to electrical energy. Possible sources of excitation are not limited to ocean waves. Moreover, the parametric pendulum has a very rich dynamical behavior [4] and has applications in various other problems, such as mechanical, electrical, MEM, optical, and other systems, Therefore, our analysis deals with the general dynamics of a randomly excited pendulum. In [2] it was also shown that a controller based on the Time-Delay Feedback control method can maintain rotational motion of the parametric pendulum, even in case of a harmonic excitation which is disturbed by white noise. Our results deal with pendulum motion in the case of excitation by a non-white Gaussian random process. Such excitation is encountered for example in real sea states.

Pendulum energy converter

We consider a physical setup for the pendulum energy converter. In order to reduce the size of pendulum energy converter design, a tri-pendulum design with three arms at equal 120° spacings is proposed in [5], where two of the arms are at a length L_2 and the third at $L_1 > L_2$. In a preliminary study [3] such an experimental rig was excited by a vertical harmonic excitation. In the following, we will use the data of this experimental rig for a numerical case study of realistic random excitation. The considered physical experimental setup consists of the tri-pendulum, whose pivot is initially restricted to move vertically by a linear motor, which is capable to generate the desired random excitation. In addition, the plane of oscillation can be tilted by the angle θ with respect to a vertical plane, such that the influence of the acceleration due to gravity g can be changed. The resulting equation of motion for the oscillation angle φ of the pendulum energy converter can be written as

$$(I_c + I_v)\ddot{\varphi} + (c + c_g)\dot{\varphi} + (d + d_g)\dot{\varphi}\,|\dot{\varphi}| + m(L_1 - L_2)\left(g\cos\theta + \ddot{f}\right)\sin(\varphi) = 0, \tag{1}$$

with moments of inertia I_c and I_v, excitation force $f(t)$, mass m, and positive coefficients c, c_g, d, and d_g. In equation (1), we consider a generator with a nonlinear function for the torque T_g due to energy generation, given by $T_g = c_g\dot{\varphi} + d_g\dot{\varphi}\,|\dot{\varphi}|$. The harvestable electrical power P_{el} of the pendulum energy converter is the product of the generator torque T_g, the angular velocity $\dot{\varphi}$, and the energy conversion efficiency η of the generator, such that

$$P_{\mathrm{el}} = \eta\, T_g\, \dot{\varphi}. \tag{2}$$

The benefit of this setup is that an equivalent secondary resonance zone response of a simple pendulum with length l could be realized by a tri-pendulum with much shorter arm lengths of L_1 and L_2 which are given by

$$L_1 = \frac{g}{2\bar{\alpha}} \pm \sqrt{\frac{g^2}{\bar{\alpha}^2} - 4\frac{g}{\bar{\alpha}}L_2 - 8L_2^2}, \quad \text{where } 0 < L_2 < \frac{g}{2(1+\sqrt{3})\bar{\alpha}}, \quad \text{and } \bar{\alpha} = \frac{m(L_1 - L_2)g}{I_c + I_v}. \tag{3}$$

Experimental setup of the pendulum energy converter

We are interested in harvesting energy from ocean waves. Therefore we assume that the pendulum energy converter is mounted on a floating body, whose degrees of freedom are limited to heave, and whose mass is much greater than the pendulum mass. Then the influence of the pendulum motion on the heave motion is negligible. Moreover, we assume linear dynamics of the floating body. The parametric excitation due to ocean waves is modeled by a Gaussian stochastic process with a given spectral density. Various spectral densities were defined to describe different sea states. Common sea spectral densities are the Pierson-Moskowitz (PM) spectrum for deep water waves and the JONSWAP spectrum for shallow water waves. The spectral density of the heave acceleration of the floating body due to the ocean wave load is given by

$$S_{\ddot{f}}(\omega) = |RAO_{\ddot{f}}(\omega)|^2 S(\omega). \tag{4}$$

Here, $S(\omega)$ is the sea state spectral density, and

$$RAO_{\ddot{f}}(\omega) = \frac{-\omega^2 \hat{f}(\omega)}{-\omega^2(M + A_h(\omega)) + i\omega B_h(\omega) + C_h} \tag{5}$$

is the response amplitude operator of the heave acceleration of the floating body with the hydrodynamic excitation force complex amplitude per wave height $\hat{f}(\omega)$, mass M, added mass A_h, hydrodynamic damping B_h, and restoring coefficient C_h. The hydrodynamic parameters $\hat{f}(\omega)$, $A_h(\omega)$, $B_h(\omega)$, and C_h can for example be determined by strip theory.

Power generation of the pendulum wave energy converter setup

As an example in this extended abstract, we consider the tri-pendulum wave energy converter, where we use the parameters $I_v = 10.32$ kg m^2, $I_c = 0.48$ kg m^2, $m = 4.7$ kg, $L_1 = 1.0$ m, $L_2 = 0.78$ m, $c = 0.1$ kg m/s, $c_g = 0.9$ kg m/s, and $d + d_g = 0$. We use a vertical cylinder as the floating body, which is described in [6]. In order to produce the highest amount of electrical energy, the pendulum has to reach rotational motion. Therefore, we need to set up the tri-pendulum, such that the probability to reach rotational motion is as high as possible for the given random excitation. We consider PM and JONSWAP sea states with arbitrary significant wave heights H_s, and different modal frequencies ω_m. The generated power for $\theta = 0$ in a PM sea state with a significant wave height of 6 m and modal frequency $\omega_m = 1.8$ rad/s is shown in figure 1. The corresponding random time series of pendulum angle φ is shown in figure 2, where a sufficient percentage of rotational motion can be observed. As can be seen from these figures, it is possible to generate a reasonable amount of energy with the proposed setup.

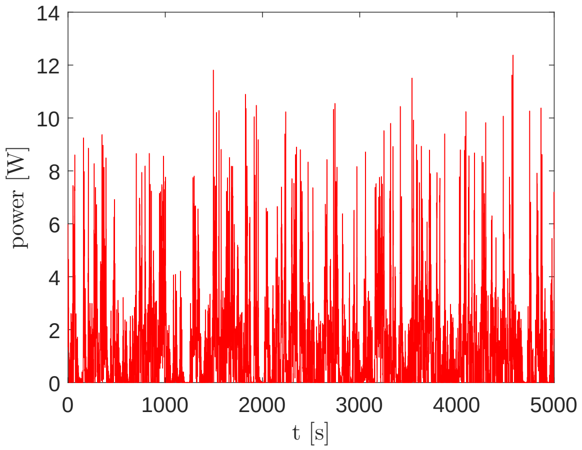

Figure 1: Harvested power over time for $\theta = 0$.

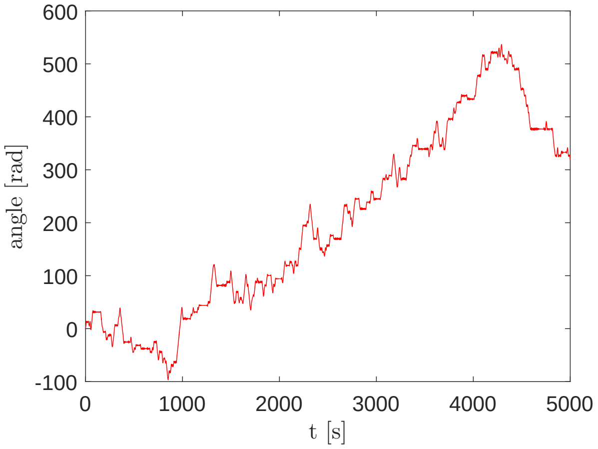

Figure 2: Random time series of pendulum angle φ for $\theta = 0$.

Conclusion

Specific results on power generation of a pendulum energy converter are obtained, which is excited by random ocean waves with PM and JONSWAP spectra. Thereby we show, that rotational motion of a specific pendulum wave energy converter design is possible even for low to moderate sea states. Results of the pendulum wave energy converter dynamics, which take into account such excitation due to random ocean waves, were not obtained before.

References

[1] Michael E McCormick. *Ocean wave energy conversion*. Courier Corporation, 2013.

[2] V Vaziri, A Najdecka, and M Wiercigroch. Experimental control for initiating and maintaining rotation of parametric pendulum. *The European Physical Journal Special Topics*, 223(4):795–812, 2014.

[3] Panagiotis Alevras, Iain Brown, and Daniil Yurchenko. Experimental investigation of a rotating parametric pendulum. *Nonlinear Dynamics*, 81(1-2):201–213, 2015.

[4] Bryan Horton, Marian Wiercigroch, and Xu Xu. Transient tumbling chaos and damping identification for parametric pendulum. *Phil. Tr. R. Soc. A*, 366(1866):767–784, 2008.

[5] Daniil Yurchenko and Panagiotis Alevras. Dynamics of the n-pendulum and its application to a wave energy converter concept. *International Journal of Dynamics and Control*, 1(4):290–299, 2013.

[6] Wanan Sheng and Anthony Lewis. Assessment of wave energy extraction from seas: numerical validation. *Journal of Energy Resources Technology*, 134(4):041701, 2012.