Control-based continuation of unstable pedestrian flows

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Summary. A scenario is studied where pedestrians have to evacuate a room with a single exit but two different route choices. The collective behaviour of the pedestrians is investigated systematically by changing the position of a triangular obstacle in front of the exit to change the preference for the two route choices. The observed bistability and hysteresis behaviour in the difference between fluxes on each side of the obstacle indicate the existence of an unstable pedestrian flow between the extreme cases where all pedestrians select the same side of the obstacle. A control-based tracking of the unstable equilibria of this flux difference is presented. This method assumes that a model for the macroscopic variable of interest is not available but the analysis is performed using the noisy output of an experiment or a numerical simulation of a microscopic particle model which does not provide high-quality derivative information.

Hysteresis behaviour of the pedestrian flux

Pedestrians want to evacuate a corridor where a triangular obstacle is the reason for a route choice left or right hand side of the obstacle (from the pedestrian point of view). Their decision depends both on the shortest way to the exit and the walking behaviour of nearby pedestrians. This route choice behaviour is investigated by changing the position of the obstacle (and thus the attractivity of each route). The so-called *social force model*, a microscopic particle model suggested by Helbing and Molnar [1] and adapted in [2] is used to describe the motion of every single pedestrian.

Hysteresis behaviour

A system with 100 pedestrians in the corridor and periodic boundary condition was numerically integrated. As a result of the up-sweep and down-sweep of the parameter μ describing the position of the triangle, two stable branches of steady flow states (see fig. 1) are observed. The jumps from one branch to the other indicate the existence of an unstable branch of steady states that connects the stable ones. The macroscopic variable of interest here is the difference of fluxes $F = F_1 - F_2$ at each side of the obstacle. Unlike [2], where the fluxes F_1 and F_2 are measured as the number of pedestrians passing the end of the obstacle per second averaged over a time interval of 10 seconds, an instantaneous flux measure f that depends on the weighted position and the velocity of all pedestrians is introduced for further numerical analysis.

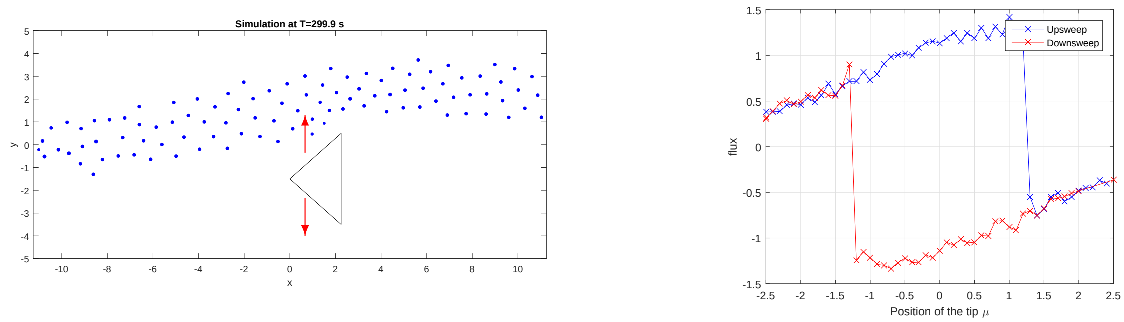

Figure 1: Left: Snapshot of the pedestrian flow. Right: Hysteresis behaviour of the instantaneous measure flux f.

Numerical continuation of steady states

In real-life problems we want to follow (continue) a branch of solutions by varying a parameter, based on the output of the experiment. While we can naturally observe the stable steady states, we use ideas from control-theory to observe the unstable ones [3], [4]. This leads to the so-called *control-based continuation*, where continuation methods can be applied directly without the knowledge of a mathematical model for the quantities of interest.

Control Based Continuation

Assume we have the output $x(t) \in \mathbb{R}^n$ of an experiment or a microsimulation that depends on a parameter μ. The evolution of the macroscopic variable of interest x is governed by the (unknown/not explicitly given) set of differential equations

$$\dot{x} = f_0(x, \mu) \tag{1}$$

that depends on the parameter μ. By varying this parameter, we can observe stable steady states of the system but not the unstable ones. We would like to feed a signal $u(t)$ to the system (1) such that the controlled system

$$\dot{x} = f_c(x, \mu) = f_0(x, \mu) + u(t) \tag{2}$$

- has the same steady states as the uncontrolled system (1)
- an unstable steady state $x = x^\star$ of (1) is stable for the controlled system (2) and thus observable
- $u(t)$ vanishes for a steady state.

If we find a point $(\hat{x}, \hat{\mu})$ such that $f_c(\hat{x}, \hat{\mu}) = 0$ and $u(t) \neq 0$ we need to make a correction to our prediction $(\tilde{x}, \tilde{\mu})$ such that $|u(t)|$ is minimized. In [3, 4] this is done be a Newton method that solves the system

$$f_c = 0 \quad , \quad u(t) = 0. \tag{3}$$

A major difficulty for this approach is that experimentally obtained data is often too noisy to provide accurate derivative information for the Jacobian of the system (3), which is essential for the implementation of the Newton scheme.

State Observer

To overcome the problem of the derivative information we determine the target $\tilde{x}$ in a different way. Inspired by [5] and [6], we consider again the system (1) for which we would like to know the unstable steady states, i.e. our goal is to locate and stabilize the unknown unstable states. For the following, y is the target state that changes in time. Here, the feedback control is $u(t) = a(y - x)$ and the controlled system results in:

$$\dot{x} = f_0(x, \mu) + a(y - x) = f_{ob}(x, \mu) \tag{4}$$

with the additional equation for the target state

$$\dot{y} = b(y - x) \tag{5}$$

where a, b are some parameters that are chosen such that the unstable states of the original system (1) become stable ones of the system (4)-(5) and thus observable. The main advantage of this method is that it does not require a derivative information.

Continuation of the Flux

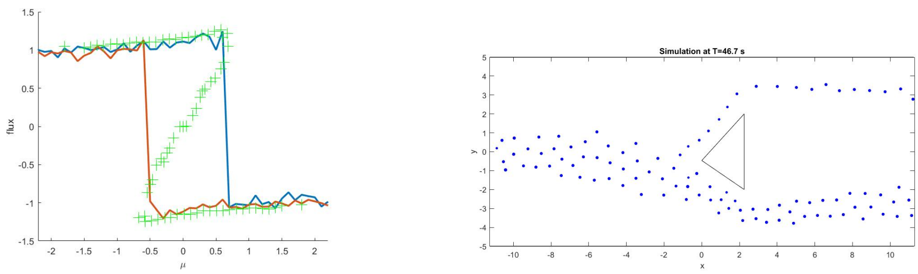

Figure 2: Left: stable and unstable branches of steady states with green crosses, obtained by the state observer method. Right: the pedestrian motion for an unstable steady state at $(f, \mu) = (-0.46, -0.6995)$.

By applying the *state observer method* in the evacuation scenario, we manage to detect the unstable branch of steady states that connects the stable branches (see fig. 2). By changing the sign of parameter b, the method also detects the (already observed) stable branches. An automatic way of changing the sign of parameter b will be developed leading to detecting the bifurcation point. Furthermore, depending on physical boundary conditions (position of the triangle, position of the walls, number of pedestrians) more unstable branches could exist which is currently under investigation.

References

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