Control of a Cart With Oscillators Under Uncertainty

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Summary. A control problem for a system, consisting of a cart with attached to it several linear oscillators is considered. The cart moves along a horizontal line under the action of a control force and unknown disturbance. The phase states of the oscillators are assumed to be not available for measuring. A bounded feedback control which in a finite time brings the cart to the prescribed terminal state from the neighborhood of it is proposed.

Keywords: linear controllable system, system of oscillators, feedback, disturbance

Statement of the problem

Consider a cart with mass m_0 moving along a straight line on a rough horizontal surface with attached to it n linear oscillators. Oscillators are assumed to be horizontally oscillating material points with masses m_i, connected with the cart via springs with spring constants $c_i,\ i = 1, 2, ..., n$. The cart is acted by a control force u and unknown disturbance v.
The motion of the system is described by equations

$$\begin{aligned} m_0\ddot{x} &= u + v + \sum_{i=1}^{n} c_i y_i \\ m_i(\ddot{x} + \ddot{y}_i) &= -c_i y_i \end{aligned} \tag{1}$$

where x describes the position of the cart on the horizontal line, y_i — elongation of the spring of the i-th spring-mass oscillator, $i = 1, 2, ..., n$. The masses and spring constants of the oscillators are assumed to be unknown and their phase coordinates and velocities $y_i, \dot{y}_i$ are not available for measuring. The disturbance v is unknown too and obeys the condition

$$|v(t)| \le V, \quad V > 0$$

A feedback control function as a function of variables $x, \dot{x}$, that meets the constraint

$$|u(x, \dot{x})| \le U, \quad U > 0 \tag{2}$$

should be designed, such that it brings the cart to the origin in a finite time. The states of the spring-mass oscillators are irrelevant at the moment when cart reaches the origin.
Several assumptions are made: control resources exceed the disturbance, initial energy of the spring-mass oscillator system is small and the cart is close enough to the terminal state at the initial time moment.

Control algorithm

To solve the above-formulated problem we use the approach, proposed in [1,2]. Let the control function be

$$u(x, \dot{x}) = -\frac{m_0 6x}{T^2(x, \dot{x})} - \frac{3m_0\dot{x}}{T(x, \dot{x})} \tag{3}$$

Here the function $T(x, \dot{x})$ is implicitly defined by the equation

$$dT^4 - 6\dot{x}^2T^2 - 24x\dot{x}T - 36x^2 = 0, \quad d > 0 \tag{4}$$

As it is determined in [1], the equation (4) has the only one positive solution for T in the whole phase space $x, \dot{x}$ except zero. This solution is given by an analytic function as the relation (4) is a polynomial equation for T with coefficients, depending on the phase variables $x, \dot{x}$. Moreover, the function $T(x, \dot{x})$ can be defined at zero as $T(0, 0) = 0$, which preserves the continuity of it.
The coefficients of the feedback control function (3) at phase variables $x, \dot{x}$ infinitely increase when these phase variables tend to zero. However, the control (3) is bounded in the whole phase space: the special choice of the parameter d in the equation (4) ensures the constraint (2).
Let

$$f(t, y) = v + \sum_{i=1}^{n} c_i y_i, \quad y = (y_1, \dots, y_n)$$

Together with the initial system (1) one may consider the following equation

$$\ddot{x} = u + f \tag{5}$$

where f is treated as an uncertain disturbance. As it is shown in [2], the derivative of the function T according to equation (5) in the absence of the disturbance (i.e. $f \equiv 0$) obeys the inequality

$$\dot{T} < 0$$

Moreover, as it is proven in [3], the derivative of the function T according to equation (5) meets the inequality

$$\dot{T} < -\gamma, \quad \gamma > 0$$

under the condition

$$|f| \leq \frac{3 - \sqrt{3}}{6} U$$

Consider the Lyapunov function

$$V(x, \dot{x}, y, \dot{y}) = T(x, \dot{x}) + \frac{1}{2} \sum_{i=1}^{n} \left(c_i y_i^2 + m_i (\dot{x} + \dot{y}_i)^2 \right)$$

This function is positive-definite and the derivative of it according to system (1) is written as

$$\dot{V} = \dot{T} - \dot{x} \sum_{i=1}^{n} c_i y_i$$

The main result of the present research is formulated in the following theorem.
Theorem. *There exists $V_0 > 0$, such that for trajectories of system (1), (2), starting in the neighborhood of the origin of the phase space $x, \dot{x}, y, \dot{y}$, given by the relation $V(x, \dot{x}, y, \dot{y}) \leq V_0$, the following inequality holds*

$$\dot{V} < 0$$

Moreover, along these trajectories

$$\dot{T} < -\sigma, \quad \sigma > 0$$

It follows from the Theorem, that the function $T(x, \dot{x})$ vanishes to zero in a finite time, i.e. every trajectory of equation (5) reaches the origin of the phase space $x, \dot{x}$ in a finite time. This means that the cart will be stopped and further be held in the origin by control (2).

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References

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