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Interacting global and slow manifolds

José Mujica, Bernd Krauskopf and Hinke M. Osinga
Department of Mathematics, University of Auckland, Auckland 1142, New Zealand

Summary. We consider the consequences of a first quadratic tangency between the unstable manifold of a saddle-focus equilibrium and a repelling slow manifold in an ordinary differential equation model with one fast and two slow variables. This phenomenon occurs near a singular Hopf bifurcation, and we show that it leads to the creation of large-amplitude excursions and associated mixed-mode periodic orbits. The local and global organisation of phase space during the transition through the tangency is obtained by computing the global and slow manifolds as families of orbits segments with a two-point boundary value problem set-up.

We are concerned with three-dimensional slow-fast systems with one fast and two slow variables. More specifically, we study the ordinary differential equation model from [2] for a singular Hopf bifurcation, given by

$$\varepsilon \dot{x} = y - x^2 - x^3, \quad \dot{y} = z - x, \quad \text{and} \quad \dot{z} = -\nu - ax - by - cz. \tag{1}$$

As was found via the integration of selected trajectories up to a suitable section [1, 2, 3], system (1) features a tangency between the unstable manifold $W^u(p)$ of a saddle focus equilibrium p and the repelling slow manifold S^r_ε. Throughout, we use the same fixed values of the parameters $a = 0.008870, b = -0.5045, c = 1.17$ and $\varepsilon = 0.01$, while ν is varied as the bifurcation parameter. We are interested here in the consequences of this tangency, both locally near p as well as globally throughout phase space. To this end, we compute the surfaces $W^u(p)$ and S^r_ε and their intersection sets with different types of sections as families of orbit segments, which are specified by suitably defined boundary value problems; see [1, 4] for details.

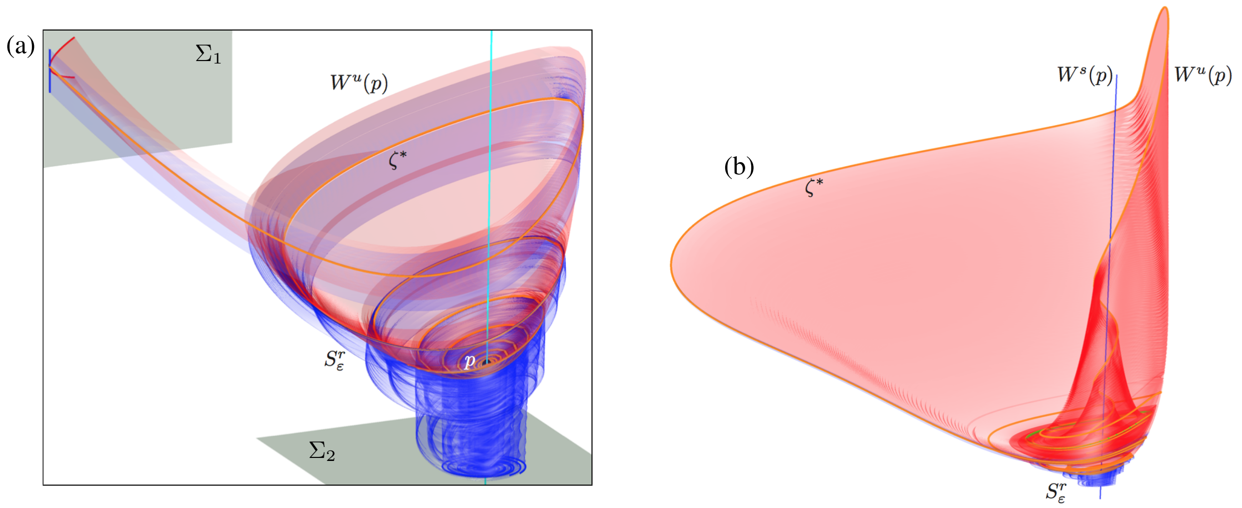

Figure 1: Local (a) and global (b) views of the first tangency between S^r_ε and $W^u(p)$.

The moment of the first tangency is illustrated in Fig. 1. Panel (a) shows the situation locally near p: the quadratic tangency is clearly visible in the section Σ_1, where the intersection sets of S^r_ε and $W^u(p)$ are a straight line and a parabola, respectively; the tangency point corresponds to a special trajectory ξ^*, which we call a connecting canard, along which the surfaces S^r_ε and $W^u(p)$ meet; all other trajectories of S^r_ε stay below the surface $W^u(p)$ and intersect the section Σ_2 in two spirals that connect at the intersection point of the stable manifold $W^s(p)$. Figure 1(b) shows the global consequence of the tangency: the surface $W^u(p)$ extends far up along S^r_ε, so that the connecting canard ξ^* leaves and then re-enters a neighbourhood of p.

The transition through the first tangency of S^r_ε and $W^u(p)$ leads to a spectacular change of the two surfaces involved: $W^u(p)$ grows dramatically in size; moreover, after the tangency, S^r_ε no longer resides only below $W^u(p)$, but also accumulates on the upper branch of $W^s(p)$. The latter leads to further quadratic tangencies between S^r_ε and $W^u(p)$, and the creation of large periodic orbits. These are characterised by large excursions interspersed with small-amplitude oscillations near p, meaning that they are a type of mixed-mode oscillations. Interestingly, the large MMO periodic orbits, which may be attracting, are actually linked with a attracting small-amplitude periodic orbit arising from the Hopf bifurcation — representing a robust mechanism for complicated recurrent dynamics.

References

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