Asymptotic study of the model of a rowing boat

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Summary. Mathematical model of motion of a rowing boat is studied. The rowing system is based on a double slider-crank mechanism. Relation between an average hydrodynamic force and an average inertia force is assumed to be sufficiently small. The Poincare-Pontryagin approach is used to describe, how an average speed of the body at steady motions depends on a control torque applied to a motor.

Introduction

A lot of applications require using waving mechanism as a working element of a floating or flying vehicle [1]. One of general advantages of a rowing boat is that it can be used in presence of water plants without regular cleaning of a screw. In this paper we suppose that the rowing element of the boat is represented by a slider-crank mechanism; the wing is attached to the link; the motor applies control torque to a shaft of the crank.

Description of the system

Assume that the rowing mechanism consicts of two parts located symetrically (that are right and left oars). Each of them is a slider-crank mechanism (fig. 1). Such mechanism consists of the crank OA (of the length *r*) and a link AB (of the length *l*). The wing is rigidly joint to the link AB. The slider B moves along the axis O*x*. The motor applyes the torque *M* to the shaft O of the crank. Assume that the other oar moves symmetrically so that the center of mass of the boat moves along the straight line. Thus, the speed of the point O is parallel to the axis P*y*.

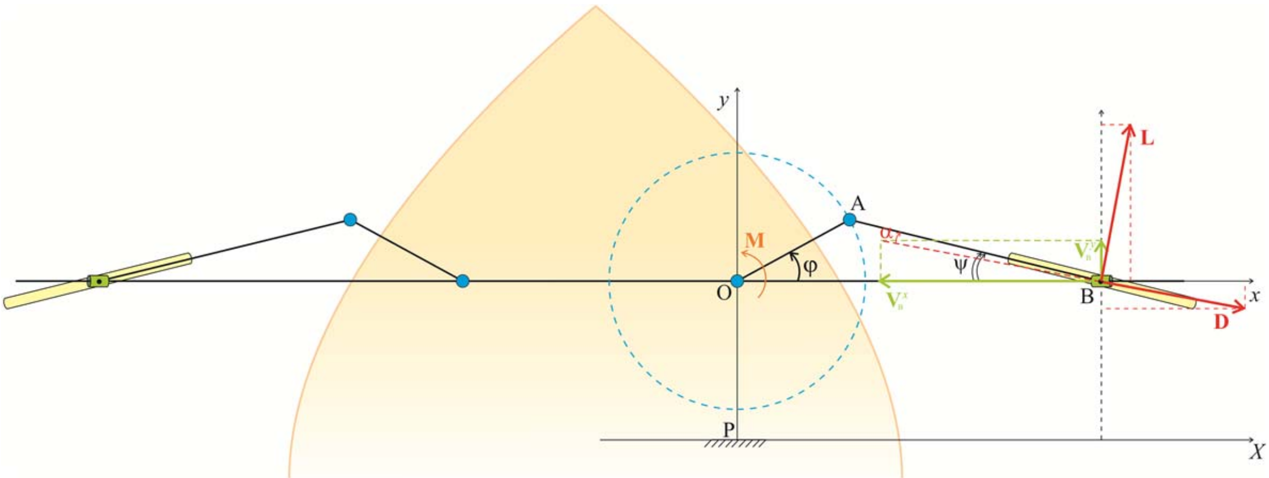

Fig. 1 The scheme of the rowing mechanism (top view).

The system has two degrees of freedom. Choose the following generalized coordinates: the angle φ between O*x* and OA and the coordinate *y* of the point O along the P*y* axis.
Suppose that hydrodynamics forces are as follows:

$$D = 0.5\rho S V_B^2 C_d(\alpha); \qquad L = 0.5\rho S V_B^2 C_l(\alpha).$$

Here ρ is the air density, S is the wing area, $C_d(\alpha), C_l(\alpha)$ are drag and lift aerodynamic coefficients, α is the instantaneous angle of attack (fig. 1).
Assume that the value $\mu = D = 0.5\rho S r^3 / J_O$ is a small parameter of the model (J_O is moment of inertia of a crank around the axis O).

Stages of the research

Equations of motion of the system were derived. Numerical integration of these equations for a wide range of parameters and initial conditions shown, that the system possesses an attracting steady regime of motion. Moreover, with some additional assumptions, conditions of existence of such regime can be obtained via the Poincare-Pontryagin approach described in [2, 3]. This approach allowed to describe, how an average speed of the point O at a steady motion depends on a value of the control torque *M* for the case $\mu \to 0$.

Numerical integration of dynamical system was performed to describe the relation between an average V_O and M for finite values of μ .

A prototype of the boat was constructed and tested. Thus, experimental dependence $\bar{V}_O(M)$ was obtained.

All results (analytical, numerical, and experimental) proved to be in qualitative agreement with each other.

Results

It is shown that, for small values of μ , the average speed V_O at a steady motion is almost proportional to a square root from the value of the control torque M. This result qualitatively agrees with experimental data.

From numerical integration, it follows that the average speed V_O is slightly sensitive to the value of the wing area S.

This work was partially supported by the Russian Foundation for Basic Research, projects NN 15-01-06970, and 16-31-00374.

References

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