Bistability of a cantilever actuated by fringing electrostatic fields

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Summary. We report on an approach to achieve bistability in a simple initially planar cantilever device. Using reduced order (RO) Galerkin and numerical finite elements (FE) analysis, we show that actuation by fringing electrostatic field combined with the direct forcing introduces a tuneable restoring electrostatic force and may bring the cantilever to a bistability threshold. The reported approach allows to achieve bistable behaviour in a cantilever device distinguished by robustness and low sensitivity to thermal and residual stress. In addition, we show, by means of the model, that the proposed operational scenario results in higher frequency sensitivity in the vicinity of the critical points of the device.

Introduction

Resonant micro/nano cantilevers are among the most sensitive devices used for detection of extremely small masses, biochemical substances, or for surface scanning by atomic force microscopy (AFM)[1]. Despite progress in their design, further improvement of performance is still required. One of the approaches is based on device operation in vicinity of instability points, where the effective stiffness is reduced and the sensitivity is maximal. Electrostatic softening near pull-in results in higher sensitivity but has a danger of pull-in collapse as its drawback. Bistable curved micro beams [2] or offset clamped beams [3] allow reversible operation near instabilities, but are sensitive to temperature and residual stress. We show that combined actuation by fringing electrostatic forces and by a direct transversal loading allows tailoring of the device response and may lead to bistable behavior. Cantilevers operated by fringing fields were reported in [4] but bistability was not mentioned. Bistable behavior of a cantilever with parallel plate electrodes was reached in [5] using close loop control. Here we report on a simple and robust bistable cantilever device ddistinguished by low thermal sensitivity.

Model

Our device is a cantilever designed to bend in the out-of-plane (z) direction, Fig. 1(a). A planar side electrode of length L_S is located at the distance g_S from the beam and is a source of the restoring electrostatic force F_R, Fig 1(b), [6]. Combination of the direct actuation by a force in a transverse z direction with the restoring mechanical and electrostatic fringing field forces, may result in an equilibrium curve corresponding to the bistable behavior, Fig. 1(c). The beam was described in the framework of the Euler-Bernoulli theory. The electrostatic forces incorporated into the RO Galerkin model were obtained by curve fitting the results of the two-dimensional (2D) FE electrostatic analysis, Fig. 1(d). Two possibilities of actuation in the transversal direction were analyzed – a parallel-plate electrode located under the beam and a thin piezoelectric layer attached to be beam. Direct numerical solution (collocation method) of the differential equation of the beam along with the three-dimensional (3D) FE analysis (Comsol) were used to verify the RO model.

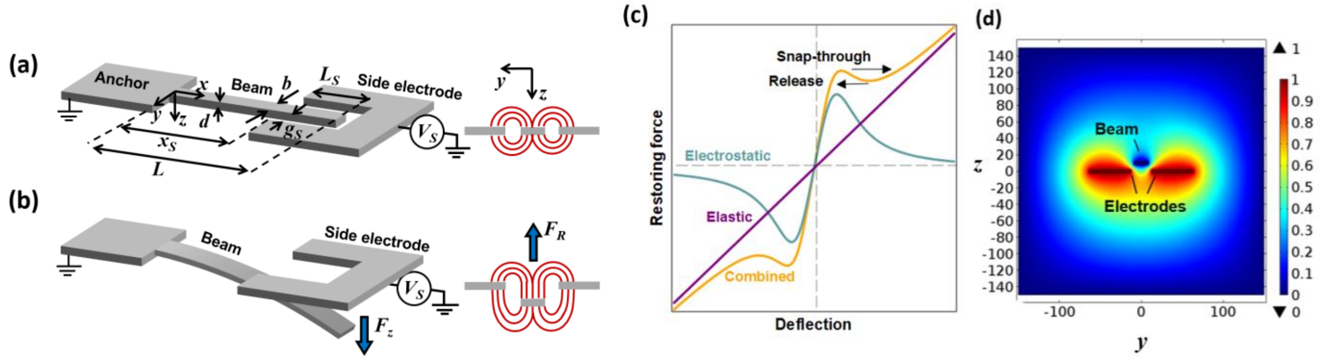

Fig. 1: (a) Schematics of an undeformed cantilever under symmetric fringing fields emerging from the side (fringing) electrode. (b) Deformed beam actuated by a transversal loading F_z and side electrodes connected to the voltage V_S. Insets show the fringing fields for each case. (c) Schematic various forces as a function of the beam's endpoint deflection: elastic mechanical (purple), fringing field (side electrode) electrostatic restoring (blue) and the divergent electrostatic from the parallel-plate (PP) electrode (orange). (d) Electric potential around the beam (grounded) and electrodes (unit voltage) in yz plane, for deformed configuration (2D FE solution).

Model Results

Calculations were carried out using L = 2000 μm long, b = 16 μm wide and d = 3 μm thick beams assumed to be made from single crystal silicon with the Young's modulus E=160 GPa. The side electrode length was L_S = 250 μm and the distance between beam and electrode was g_s = 5 μm. Equilibrium curves for the generic case of a linear uniform transversal force f_z is shown in Fig 3(a). Curves corresponding to V_s =50 V and V_s =90 V, Fig. 3(a), show typical behavior below and above the bistability threshold, respectively. An analytical expression obtained for the critical threshold value of the voltage predicted $V_s^{CRIT} = 62$ V. In contrast to bistable offset structures [3,4], our cantilever-type device demonstrates snap-through jump while actually moving farther away from the electrode and not toward the electrode. The dependence of the linearized natural frequency of the beam on the transversal loading f_z is shown in Fig. 3(b) for differing values of V_s. To be operated as a sensor the following scenario is suggested. For V_s slightly below the bistability threshold V_s^{CRIT}, pre-loading of the beam by the parallel-plate electrode or piezoelectric layer brings the cantilever to the configuration at the verge of the snap-through instability (blue line in Fig. 3(a)). In this case the equilibrium curve contains an inflection point where the slope of the curve can be tailored to be low and the effective stiffness and the natural frequency of the device are reduced. In the vicinity of the inflection point the sensitivity of the natural frequency to loading is significantly higher than in an undeformed cantilever. On the other hand, in contrast to the case of $V_s > V_s^{CRIT}$ the beam does not exhibit snap-through collapse (purple lines in Fig. 3) and smooth continuous operation can be achieved.

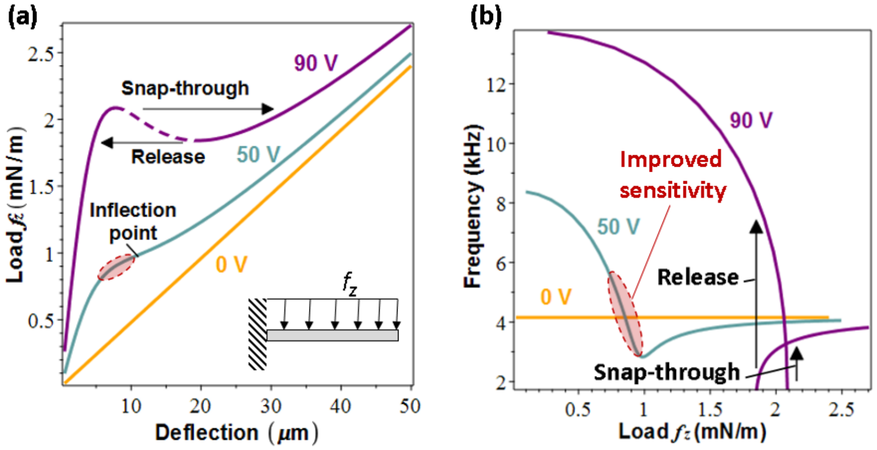

Figure 3: (a) Equilibrium curves for the generic case of a loading by a uniform transversal force f_Z and different values of the side electrode voltage V_S (numbers). The curve corresponding to V_S=90 V reflects bistable behavior. Solid lines correspond to the stable solution, dashed line depicts unstable branch of the equilibrium path. (b) Fundamental mode frequency of the cantilever as a function of the transverse "mechanical" loading f_Z. Different curves correspond to the different values of the voltage V_S (numbers) applied to the side electrode. Discontinuity can be seen on the curve corresponding to V_S = 90 V which corresponds to the bistable behavior.

References

[1] Boisen A, Dohn S, Keller S S, Schmid S and Tenje M 2011 Cantilever-like micromechanical sensors *Reports Prog. Phys.* **74** 36101

[2] Krakover N, Ilic B R and Krylov S 2016 Displacement sensing based on resonant frequency monitoring of electrostatically actuated curved micro beams *J. Micromechanics Microengineering* **26** 115006

[3] Su J, Yang H, Fay P, Porod W and Bernstein G H 2010 A surface micromachined offset-drive method to extend the electrostatic travel range *J. Micromechanics Microengineering* **20** 15004

[4] Small J, Irshad W, Fruehling A, Garg A, Liu X and Peroulis D 2012 Electrostatic fringing-field actuation for pull-in free RF-MEMS analogue tunable resonators *J. Micromech. Microeng* **22** 95004–10

[5] Seleim A, Towfighian S, Delande E, Abdel-Rahman E and Heppler G 2012 Dynamics of a close-loop controlled MEMS resonator *Nonlinear Dyn.* **69** 615–33

[6] Linzon Y, Ilic B, Lulinsky S and Krylov S 2013 Efficient parametric excitation of silicon-on-insulator microcantilever beams by fringing electrostatic fields *J. Appl. Phys.* **113** 163508