Dance-like motions in optimal walking

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Summary. Gait generation for bipedal robots via gradient-based optimization may result in local optimality of the solution. We have applied a gradient-based energy optimization method to a seven degree of freedom robot model with rolling feet. The convergence to several solutions in a wide speed range proves the coexistence of local optima with more or less similar energy expenditure. The optimal gait depends on the average walking speed: there are three different stable gaits which become optimal for slow, medium or fast speeds, respectively. An examination of several gaits reveals dance-like motions which indicate some optimality in human dancing.

Introduction

Gait generation for bipedal walking has been of interest in the areas of biomechanics [1] and robotics [2] for decades. While the objective in biomechanical studies is often to match experimental data from human subjects [1, 3], gait generation for robotic applications can also be performed via numerical optimization [4, 5, 6]. On the one hand, these gaits can be used in real-time control [5, 6], on the other hand, simulations and optimization themselves can be used for parameter studies.

In this contribution, gait generation via numerical optimization for a robot model with round feet (cf. Fig. 1) is investigated. Energy efficiency, a widespread criterion [4, 5, 6, 7], is maximized via gradient-based optimization for a range of speeds. Local optima are tracked by a path-following approach and the results are discussed.

Model and Optimization

The planar robot model in Fig. 1 which consists of seven rigid bodies (the trunk, two thighs, two shanks and two feet) connected by actuated ideal revolute joints in the hip, the knees and the ankles is investigated. Its dimensions and mass distribution are approximated via human parameters [8]; the foot geometry is fitted to experimental data from [9]. Walking gaits on even terrain are simulated which consist of a single support phase – only the stance foot is on the ground – and an instantaneous double support phase – the former swing foot touches the ground in an ideally plastic impact and the other foot lifts-off without interaction. Gait generation is realized by a trajectory tracking controller using the hybrid zero dynamics approach for rolling feet [6]. Due to the robot's underactuation, the control inputs $\mathbf{u}$ (the joint torques) cannot affect the robot's absolute orientation θ directly. In the reference motion without control deviation – the controller's zero dynamics – the joint angles $\mathbf{q}_\mathrm{j}$ match the state dependent reference trajectories

$$\mathbf{q}_\mathrm{j} = \mathbf{h}_\mathrm{r}(\theta) = \sum_{k=0}^{M} \boldsymbol{\alpha}_k \binom{M}{k} s^k (1-s)^{M-k} \,, \qquad s = \frac{\theta - \theta^+}{\theta^- - \theta^+} \in [0,1] \,, \tag{1}$$

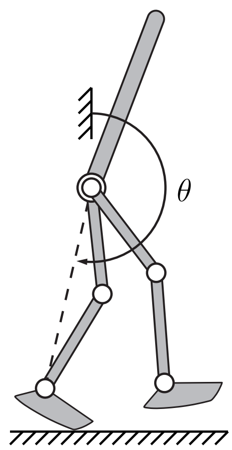

Figure 1: Robot model

which are defined as Bézier polynomials of degree M with parameters $\boldsymbol{\alpha}$ and the absolute angle θ^+ (θ^-) at the beginning (end) of the step. The parameters $\boldsymbol{\alpha}_\mathrm{i} = [\boldsymbol{\alpha}_2, \ldots, \boldsymbol{\alpha}_M]$ are chosen to be independent and the remaining dependent parameters $\boldsymbol{\alpha}_\mathrm{d} = [\boldsymbol{\alpha}_0, \boldsymbol{\alpha}_1]$ are subsequently calculated from $\boldsymbol{\alpha}_\mathrm{i}$ and the periodicity conditions imposed by the plastic impact, cf. [5, 6]. This results in the hybrid zero dynamics – zero dynamics which are invariant with respect to the impact. Due to the state dependent definition of $\mathbf{h}_\mathrm{r}(\theta)$, the hybrid zero dynamics correspond to a mechanical system with one degree of freedom. There is a semi-analytic solution via quadratures [5] for the hybrid zero dynamics, which is subsequently used to calculate the corresponding joint torques $\mathbf{u}$ via inverse dynamics.

The joint actuators are modeled as electric motors with the electrical energy input at each joint $P_{\mathrm{el},i} = c_\mathrm{stat} u_i^2 + u_i\, q_{\mathrm{j},i}$, $i = 1 \ldots 6$. With the motor constant c_stat from [7], $c_\mathrm{stat} u_i^2$ are the i-th motor's heat losses and $u_i\, q_{\mathrm{j},i}$ its mechanical power. Taking only the supplied electrical power into account, the robot's *cost of transportation*

$$c_\mathrm{T} = \frac{\sum_{i=1}^{6} \int_0^T \max(P_{\mathrm{el},i}, 0)\,\mathrm{d}t}{mgL} \tag{2}$$

is defined as consumed energy during one step with duration T, divided by the robot's weight mg and its step-length L. Energy optimal gaits are generated via minimization of c_T considering the following model assumptions as inequality constraints $\mathbf{g}_\mathrm{ineq} \leq \mathbf{0}$: unilaterality of ground contacts, stiction, no knee hyper-extension; and one equality constraint for the average walking speed $\bar{v}$: $g_\mathrm{eq} = 0 = \bar{v} - \frac{L}{T}$. The corresponding optimization problem is

$$\min_{\boldsymbol{\alpha}_\mathrm{i}} \; c_\mathrm{T}(\boldsymbol{\alpha}, \bar{v}) \qquad \text{subject to} \quad \mathbf{g}_\mathrm{ineq} \leq \mathbf{0} \,, \; g_\mathrm{eq} = 0 \,. \tag{3}$$

Local minima for this constrained optimization problem are calculated via sequential quadratic programming (SQP) in *Matlab*. Subsequently, the local minimum is tracked by a path-following approach: as trivial prediction step, the solution for the average speed $\bar{v}_\ell$ is used to initialize the correction step for the next speed $\bar{v}_{\ell+1}$ – optimization via SQP-algorithm. Small variations of $\bar{v}$ result in tracking of the local minima, while large variations may result in convergence to a different solution – if the initial minimum is not the global one for the corresponding speed $\bar{v}$.

Results and Conclusions

Figure 2 depicts the optimization results which were found and tracked for a robot with an additional torsion spring coupling its thighs. Four different solutions, corresponding to four different gaits, are distinguished by different colors. Only two of the solutions exist in the whole speed range of $\bar{v} \in [0.3, 2.3]$ m/s. Gait 1 exists only for $\bar{v} \geq 1.4$ m/s, gait 2 only for $\bar{v} \leq 1.6$ m/s. Furthermore, gait 4 is unstable for $\bar{v} < 2.0$ m/s which is indicated by the dashed line. The spring stiffness between the robot's thighs has a strong influence on the robot's optimal walking dynamics and strongly influence the limits for the existence of local solutions.

The four gaits correspond to different local minima of the optimization problem which are not connected to each other. For very slow speeds $\bar{v} < 0.5$ m/s, gaits 2, 3 and 4 have a very similar cost of transportation. Furthermore, the lines of gaits 2 and 3 cross each other at $\bar{v} < 1.1$ m/s. The coexistence of two solutions with the same cost of transportations can only occur, if the local minima are not connected. Gait 2 is optimal at slow speeds $\bar{v} < 1.1$ m/s, gait 3 at medium speeds $1.1 < \bar{v} < 2.0$ m/s and gait 4 becomes the optimal stable gait at high speeds.

Tracking of local solutions by the predictor-corrector path-following approach allow for an investigation of the different local solutions and changes in the walking dynamics which distinguish them from each other. A comparison of gaits 2 and 3 at $\bar{v} = 1.6$ m/s, the limit of the path-following algorithm for solution 2, reveals that gait 3 includes a rocking motion of the stance foot at the beginning of the step which resembles a motion from samba dancing that is unincisive in gait 2.

For lower spring stiffnesses between the thighs, the limit for existence of gait 1 is shifted to slower speeds, where the cost of transportation is significantly higher than for the other gaits. Compared to the other gaits, gait 1 distinguishes itself by larger steps and longer step durations. Because the step-length does not significantly decrease with the average walking speed, the step duration is respectively increased. This leads to a gait which resembles the Feather step, a dance figure in slowfox dancing.

These examples illustrate emergent dynamics in optimal bipedal walking due to the model parameters (geometry and stiffness) and the objective of the optimization. A model with point feet and similar parameters does not result in co-existent local solutions using the same objective, cf. [7]. The discussed resemblance of dance steps and individual gaits corresponding to local optima indicates that there might be some optimality in human dancing.

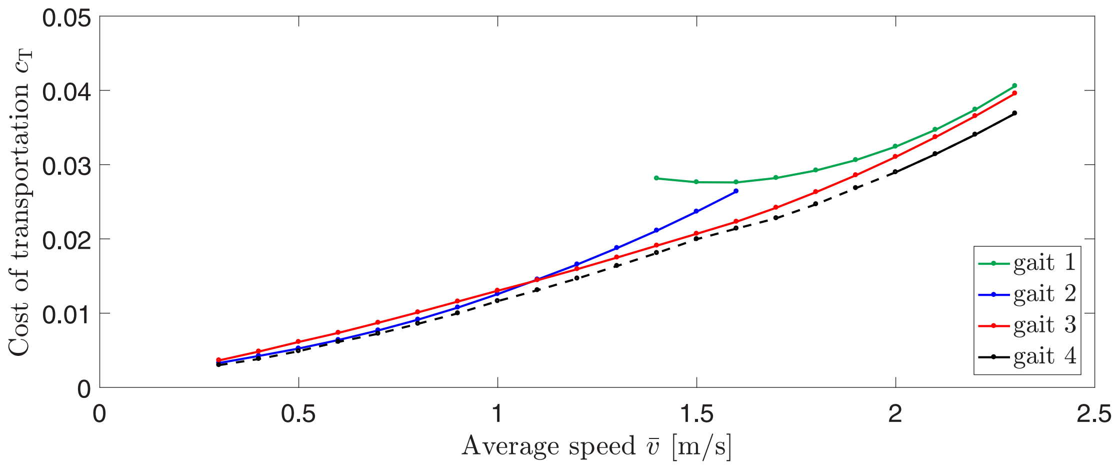

Figure 2: Cost of transportation c_T for the four gaits. Dashed line indicates unstable solutions for gait 4 and $\bar{v} < 2.0$ m/s.

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