Nonlinear analysis of the body sway of car-trailer combinations with nonlinear shock absorber and tire characteristics

Ning Zhang*, Jian Ma*, Tian Mi* and Guo-dong Yin*

**Vehicle Engineering Laboratory, School of Mechanical Engineering, Southeast University, Nanjing* 211189*, China*

Summary. The shock absorbs installed in vehicle suspensions have some nonlinear properties. And tires have strong nonlinear characteristics in vehicle system dynamics. Their combined influence on dynamic stability of car-trailer combinations are investigated in this paper. The bifurcation analysis can reveal this influence based on a theoretical extended single-track model for car-trailer combinations, in which modules describing the both nonlinearities are integrated. The body sway phenomenon might be explained by the corresponding nonlinear analysis. In this way, the influence of the nonlinear shock absorber characteristic on the dynamic stability of car-trailer combinations can be studied when the nonlinear tire characteristic is also considered.

Introduction

Every car-trailer combination (CTC) has a dynamic critical speed v_{crit} (ca. 100 km/h) by which the system dynamic stability boundary is determined. However, the v_{crit} of a normal passenger car is relatively too high (above 250 km/h) and its dynamic stability problem can be neglected in normal driving situations. A trailer introduces one more pair of system conjugate characteristic roots by which the v_{crit} is redefined and decreases significantly, while the normal pair of system conjugate characteristic roots, same as that of a normal passenger car, is not crucial to its dynamic stability.
This dynamic critical speed v_{crit} is a nature for a CTC. It has a close relationship with the system yaw damping ratio. The v_{crit} indicates a zero yaw damping ratio with a steady state yaw oscillation in the horizontal plane. Such a system under its critical stable state is very sensitive to any disturbance. Therefore, the potential dynamic instability is a challenging problem in the stability investigation of car-trailer combinations (CTCs).
The research on this topic can be traced back to 1960s. Kurtz and Anderson [1] presented a state-of-the-art survey on the handling characteristics of CTCs in that time period. Theoretical fundamentals were established at this stage. Then several analytical models, which contained from 3-DOF to 24-DOF [2,3], were developed and with commercial software highly sophisticated system models were available. Later Sagan [4] performed a comprehensive study on the handling characteristics of CTCs and the sensitivity of some system parameters to the dynamic stability were achieved as well. However, they were achieved with a simple linear model in the horizontal plane and influence from the vertical dynamics was not considered.

Nonlinear tire characteristic

The tire characteristic is nonlinear in practice and plays one of the most important roles in determining system stability. It is popularly simplified as a linear module at the first step. However, its nonlinear model, like the Magic formula tire model, is frequently adopted once the tire nonlinear range has also to be considered. In this situation, nonlinear analysis methods can explain more about the system properties, particularly for complex systems, like CTCs. Yang et al. [5] contributed some interesting results based on the Hopf bifurcation analysis at the system equilibrium points when the longitudinal velocity v_x locates in 15-30 m/s. Stépán's group [6] investigated the same problem of CTCs with a time-delayed nonlinear tire model under 2 m/s. But the knowledge about the influence from the vertical dynamics is still missing.

Nonlinear shock absorber characteristic

Because of the complexity and less practical demand for normal passenger cars, few contributions are found on the topic of dynamic coupling between the horizontal dynamics and vertical dynamics, especially for CTCs. Most studies are performed only in the horizontal plane and normally the horizontal dynamics and vertical dynamics are modeled and analyzed separately [4]. Recently Bedük et al. [7] have published a research on "Effect of damper failure on vehicle stability" and point out that damper failure might lead to vehicle instability in some critical driving situations without ESP control. But no contribution is conducted on this aspect for CTCs.

Dynamic stability of car-trailer combinations

Considering the stability of vehicle systems, there are two different types of instability problems in the horizontal plane: static instability and dynamic instability [8]. The static instability is also called divergent instability, in which the typical system state variables increase without experiencing oscillations. This type of static instability is well known as jack-knifing, especially common for articulated vehicles. But a statically stable system with a big oscillatory response is also difficult to control, like a CTC under its critical stable state. This nature is named as dynamic instability, known as snaking or sway [1]. While the system steady state behavior is mainly considered in static stability, its transient behavior is considered more in dynamic stability. The shock absorber's behavior is dynamic in nature and hence it influences system dynamic stability all the time as long as yaw oscillation exists. Therefore, this research is particularly beneficial to the investigation of the dynamic stability of CTCs.

Objectives

The combined influence of the nonlinear shock absorber and tire characteristics on dynamic stability of CTCs is the focus here. They both have a practical piecewise-linear property. Both nonlinearities do have influences on the dynamic stability of CTCs, although the nonlinear tire characteristic is relatively strong compared with the other. Therefore, their combined influence is an ultimate goal to understand the influence of the nonlinear shock absorber characteristic of the towing car in practice.

Analysis methods and results

A methodology comprising the theoretical study and experimental study is applied in this paper. Because the system is nonlinear, nonlinear analysis methods are necessary. It is already found that a CTC with nonlinear tires in its critical cornering situations shows dynamic instability indicated by some saddle-node bifurcation or Hopf bifurcation [5]. Therefore, bifurcation analysis is considered as a main tool in the theoretical analysis based on an extended single-track model (ESTM) of CTCs (see Fig. 1). Both nonlinearities are described by the Magic formula model (see Fig. 2). Besides, a series of road tests are designed and performed according to the lateral stability test standard ISO 9815. A car-caravan combination is applied in road tests.

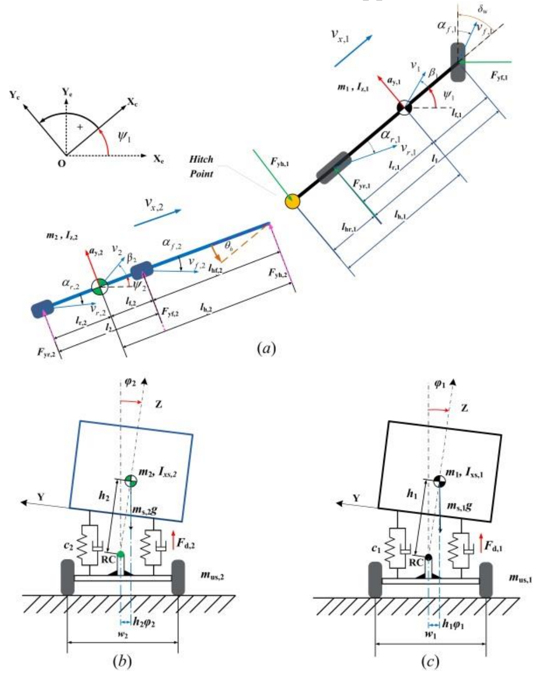

Fig. 1: (*a*) An extended single-track model for a CTC with a tandem-axle trailer in horizontal plane;
(*b*) Trailer model in vertical plane;
(*c*) Towing car model in vertical plane

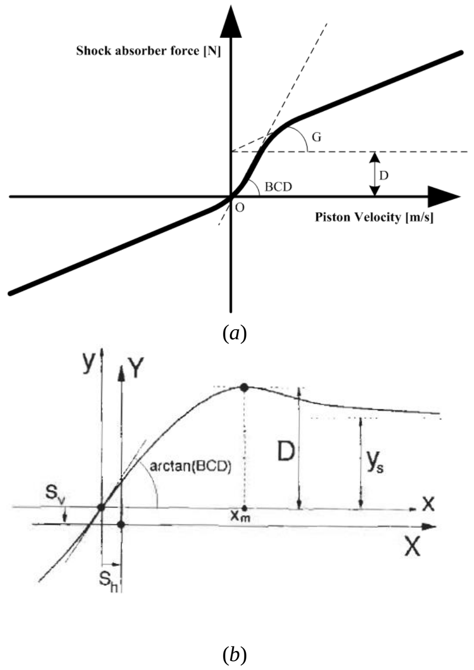

Fig 2: (*a*) Magic Formula damper model;
(*b*) Magic Formula tire model

Conclusions

It is found that no significant difference exists in the v_{crit} between the hard shock absorbers and the soft shock absorbers installed in the towing car in road tests. The experimental study reveals that change of the damping coefficient has hardly any influence on the system dynamic stability. However, dynamic coupling between the horizontal dynamics and vertical dynamics for CTCs is still unclear. Besides, whether the influence of the damping coefficient is hidden or counteracted by the combined influence of two nonlinearities is still not confirmed. The research result can serve as a guideline for a robust damper design considering the variations of its structural components and the stability of CTCs.

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