Non-classical nonlinear normal vibration modes in mechanical systems

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Summary. Nonlinear normal modes (NNMs) of forced *chaotic vibrations* can be found in models which are obtained by digitization of some elastic systems that have lost stability under external compressive force. A system of non-autonomous Duffing equations can be obtained; chaotic motions appear as the force amplitude is slowly increased. A stability of periodic or chaotic vibration mode in a space with a greater dimension is studied. In non-ideal dissipative systems NNMs by Kauderer-Rosenberg cannot be realized due to exponential decrease of vibration amplitude. In such systems under the resonance conditions the *transient nonlinear normal modes* (TNNMs), which exist only for some levels of energy, can be found. These TNNMs temporarily attract other motions of the system near values of time, corresponding to the mentioned energy levels.

Introduction

Nonlinear normal modes (NNMs) can be found in free and forced dynamics of different nonlinear mechanical and engineering systems. The concept of NNMs was first proposed for conservative systems. Theory of NNMs for conservative and non-conservative systems and different applications of this theory in engineering problems are presented in numerous publications, in particular, in [1-3]. The NNMs concept can be used not only for periodic vibrations, but for chaotic in time vibrations too. Such vibration modes can be found in dynamics of elastic systems that have lost stability under external compressive force. An appearance of new equilibrium positions except the trivial equilibrium gives a rise of the region in the system parameter space where such modes with chaotic in time behavior are observed [4].
Systems with a limited power-supply (or non-ideal systems) are characterized by the interaction of an energy source and an elastic sub-system which is under action of the source. For these systems the external excitation is is dependent on coordinates of the elastic sub-system. The most interesting effect appearing in these systems is the Sommerfeld effect [5], when the stable resonance regime with large amplitudes appears in the elastic sub-system. Resonance dynamics of systems with a limited power-supply was first described by Kononenko [6]. Next investigations on the subject are presented in different papers, in particular, in the review [7].
Here we consider some mechanical systems which permit non-classical nonlinear normal modes. It means, in the first place, NNMs of chaotic vibrations which appear in super-critical dynamics of elastic systems. Besides, we present the transient normal modes in non-ideal dissipative systems. These modes exist only for some values of the system energy, but they are attractive near the time corresponding to these energy levels.

Nonlinear normal modes of chaotic vibrations in shell dynamics

One considers some elastic systems that have lost stability under a constant compressive force and external periodic ecxitation. In particular, the beam bending vibrations within framework of the Kirchhoff hypothesis and the dynamics of shallow-shell cylindrical shell described by the Donnell equations [8] are considered. Then the digitization by the Bubnov-Galerkin procedure is used. If displacements of the nonlinear elastic system are approximated by a single harmonic of the Fourier series expansion for space coordinates, a system having a single degree of freedom is obtained. Behavior of the model described by the non-autonomous Duffing equation was examined in numerous publications. Chaotic motions begin as the force amplitudes are slowly increased [9]. If two harmonics of the Fourier series for space coordinates are used, one obtains a set of two second order ODEs, coupled in nonlinear terms only. Two NNMs, which are determined by smooth trajectories in the system configuration place, exist here. One of these modes can be chaotic in time in some region of the system parameters. Boundaries of the region are determined as some combination of the external amplitude and parameters of nonlinearity and dissipation.
The energy ‘‘pumping’’ from one vibration mode to another one is possible. Thus one can formulate a problem of the stability of periodic or chaotic vibration mode in a space with a greater dimension. Stability of regular or chaotic modes of nonlinear beams and shells are considered. The orbital stability of trajectories of these modes is determined by the numerical-analytical approach which is based on the known Lyapunov definition of stability [4].
One considers the nonlinear beam dynamics. In Figs. 1,2 the variable y_1 represents behavior in time along the NNM

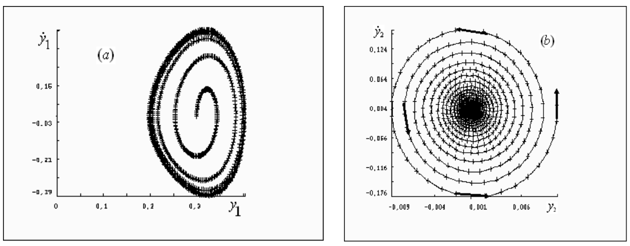

Fig.1. The unstable mode of regular vibrations.

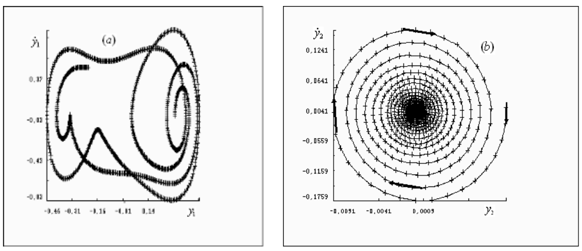

Fig.2. The stable mode of chaotic vibrations

trajectory, where this mode corresponds to the first harmonic of the Fourier series expansion for space coordinates. The variable y_2 describes variations which are orthogonal with respect to the NNM trajectory. We can see the unstable NNM which is regular in time (Fig.1, where variations increase) and the stable NNM which is chaotic in time (Fig.2, where variations tend to zero).

Transient nonlinear normal modes in non-ideal dissipative systems

The non-ideal system coupled with the Duffing-type nonlinear absorber (Fig.1) is considered under resonance conditions. Namely, the cases of external resonances on natural frequencies and both external and internal resonances are analyzed. The multiple scales method and transformation to the reduced system [10-12] stated with respect to the system energy, an arctangent of the vibration amplitudes ratio ψ and the phase difference φ are used. Analysis of the reduced system permits to describe the transfer from unstable vibration modes to stable ones, to obtain conditions of the energy localization and to obtain the principally new vibration regimes. Namely, so-called *Transient nonlinear normal modes* (TNNMs) [11,12] can be described. These TNNMs are realized only for some levels of the system energy. In vicinity of time, corresponding to these energy values, the NNNM temporarily attracts other motions if the system; then the energy decreases and the system motions tend to other stable vibration mode. The loops of the trajectories in the place (φ,ψ) (Fig. 3) correspond to the TNNM existing in the case of external resonance on the one of fundamental frequencies. When the TNNM disappears, the trajectories tend to straight line representing the stable localized NNM. Fig. 4 illustrates two equilibrium states of coupled vibrations in vicinity of simultaneous external and internal resonances; one of them is unstable but temporarily attractive TNNM, other one is stable and attractive.

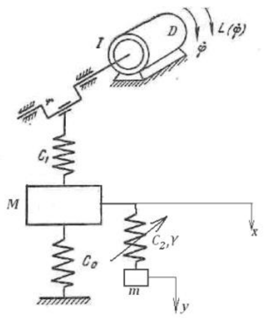

Fig. 1: The non-ideal mechanical system with the nonlinear absorber

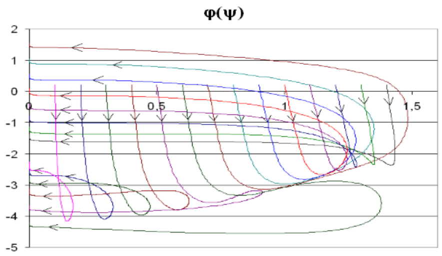

Fig. 3: Trajectories for the external resonance case

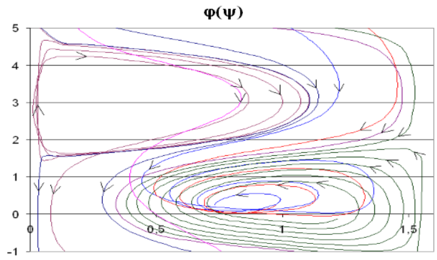

Fig.4. Trajectories for the simultaneous external and internal resonances

Conclusions

Non-classical NNMs, namely, normal modes of chaotic vibrations and the transient nonlinear normal modes can be found and described in some mechanical systems. In particular, the normal modes of chaotic vibrations are observed in the post-buckling forced dynamics of cylindrical shells that have lost stability under a constant compressive force. It can be also found in dynamics of other mechanical, or engineering systems, having few equilibrium positions. The transient nonlinear normal modes are found in the resonance dynamics of the non-ideal dissipative systems. Although these vibrations modes exist only for some levels of energy, they play an important role in the sytem transient because they attract other motions near the time, corresponding to these energy levels. Such modes can be found in other nonlinear dussupative systems under resonance conditions.

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