# Robust measurement of backbone curves of a nonlinear piezoelectric beam

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Summary. The characterization of the nonlinear frequency response of a system and the estimation of electro-mechanical coupling factors are important aspects in the framework of nonlinear vibration energy harvesting. Both problems can be addressed by measuring the backbone curve of a nonlinear mode of the system, which is defined as the amplitude-frequency relation of the free oscillations along the mode. A recent method for measuring backbone curves using Phase-Locked-Loops is presented and used in this study to characterize a nonlinear piezoelectric cantilever beam used as an energy harvester.

## Introduction

Nonlinear effects are an important aspect to consider in the framework of vibration energy harvesting. The bimorph piezoelectric cantilever beam considered in this study constitutes a simple nonlinear energy harvester [1]. It is important to understand the dynamics of such a system, therefore to characterize its nonlinear behavior in order to, for example, identify the parameters of a reduced-order model. Moreover, the Modal Electro-Mechanical Coupling Factor [2], which can be retrieved by measuring the eigenfrequency of the piezoelectric beam in open and short-circuit, is also an indicator of the performance of the system. Both the nonlinear characterization and the estimation of a coupling factor can be done by measuring the backbone curve of a single nonlinear mode.

## Nonlinear mode identification

Nonlinear modes (NLM) are a useful tool for the interpretation and the computation of the dynamics of a nonlinear systems. NLMs of conservative systems can be viewed equivalently as families of periodic solutions and invariant manifolds of the phase space [3]. This last property, which stands that a motion initiated on a NLM remains on this NLM without any transfers to the others, enables to compute them as invariant Duffing like nonlinear oscillators of the form [4]:

$$\ddot{q} + \omega_0^2 q + \Gamma_1 q^3 + \Gamma_2 q \dot{q}^2 = 0, \tag{1}$$

where $\omega_0$ is the eigenfrequency and $\Gamma_1$ and $\Gamma_2$ are two cubic nonlinear stiffnesses. The solution of Eq. (1) writes $q(t) = a\cos(\omega_{\text{nl}} t + \varphi)$, defining the so-called backbone curve in terms of amplitude-frequency relation, a first order perturbative calculation giving $\omega_{\text{nl}} = \omega_0(1 + \Gamma_0 a^2)$, where $\omega_{\text{nl}}$ is the nonlinear frequency depending on the amplitude $a$, and where $\Gamma_0 = (3\Gamma_1 + \omega_0^2\Gamma_2)/8\omega_0^2$. The easiest nonlinear mode identification then consists in determining $\omega_0$ and $\Gamma_0$ by measuring the backbone curve. It must be noticed that Eq. (1) applies at first order to any system with quadratic and cubic nonlinearities and is thus completely generic. If softening nonlinear modes are under concern, a negative coefficient $\Gamma_0$ has to be identified. Moreover, even if both nonlinear cubic terms naturally appear in Eq. (1) (with the normal form approach described in [4]), it is not possible at first order to distinguish independently the effects of $\Gamma_1$ and $\Gamma_2$ since they are merged into $\Gamma_0$. Consequently, identifying solely $\Gamma_0$ and using it into a standard Duffing oscillator with cubic term $\Gamma_0 q^3$ seems the most efficient practical identification strategy.

## Amplitude and phase resonances

Let us now study the forced oscillations of the system (1), under the harmonic forcing $F\cos\Omega t$ and adding a linear dissipative term $2\mu\dot{q}$. Frequency response curves for different forcing amplitudes are computed using the HBM method and the software MANLAB [5], and plotted in Fig. 1(left). It also displays three other curves: the phase resonance, which corresponds to the backbone curve and for which the displacement is in phase quadrature with the excitation; the amplitude resonance, which is the locus of the maxima of amplitude of each forced frequency response; the curve following the fold point, delimiting the jump-down and the unstable region. This plot shows that for a damped system, the backbone curve is distinct and slightly below the amplitude resonance curve. It is also above the fold locus curve and thus lies in the stable part of the forced response. For lightly damped systems, the difference between phase and amplitude resonance is negligible.

## Experimental methods for backbone measurements

The easiest way to obtain an experimental backbone curve is to measure several frequency responses for different vibration amplitudes and to find the amplitude resonance as the locus of the maxima of amplitude of each response. A better estimation can be obtained by the resonant decay method [6], consisting in setting the system at phase resonance for a given mode, releasing it and measuring the free oscillations. The estimation of the instantaneous amplitude and frequency allows one to obtain the backbone curve. Another recent technique is the control-based continuation (CBC) [7, 8], that combines a stabilizing feedback control and a path-following technique, as in numerical continuation methods. The method used in this study consists in tracking the backbone curve by locking the system at the phase resonance using a Phase-Locked-Loop (PLL), as it was recently done in [9, 10]. The idea behind the PLL is to generate an harmonic signal tuned to minimize a phase difference between excitation and reference signals. We use a simple and robust PLL design proposed in [11] and already implemented in [10] for the tracking of backbone curves. It consists in a phase detector, a lowpass filter, a PI-controller and a Voltage-Controlled Oscillator (VCO). Measuring a backbone curve can be done by setting a low excitation amplitude, waiting for the PLL to adjust to the phase resonance and increasing the amplitude step by step, while recording the displacement. Retrieving the amplitude of the first harmonic of the displacement is then done by homodyne detection.

## Measurement of a piezoelectric cantilever beam

The measurement method is applied to a piezoelectric bimorph cantilever beam, that can be used for energy harvesting purpose. The bimorph is constituted by a steel beam of dimensions 120 $\times$ 20 $\times$ 0.75 mm on which is glued a PZT ceramic PIC155 with dimension 20 $\times$ 60 $\times$ 0.5 mm. The beam to be measured is base-excited by an electromagnetic shaker (B&K 4808) and a power amplifier (B&K 2712). The base acceleration and the beam velocity are measured using an accelerometer (PCB 352C65) and a laser vibrometer (Polytec PSV-400). The setup is driven by a dSPACE 1104 card on which is implemented the control scheme. The frequency responses for the bimorph beam in open and short-circuit around the first resonance are obtained for several vibration amplitudes. They are plotted on Fig. 1(right). The tracked backbone curves for the two configurations are also plotted. For a given configuration (open or short-circuit), the estimated backbone is very accurate and matches the resonance amplitude, which is understandable since the damping is low. It is clear that the nonlinearity for the first mode is softening. The resonance shift between the two configurations is noticeable and quasi-constant when amplitude increases. It allows to compute a Modal Electro-Mechanical Coupling Factor of 19%.

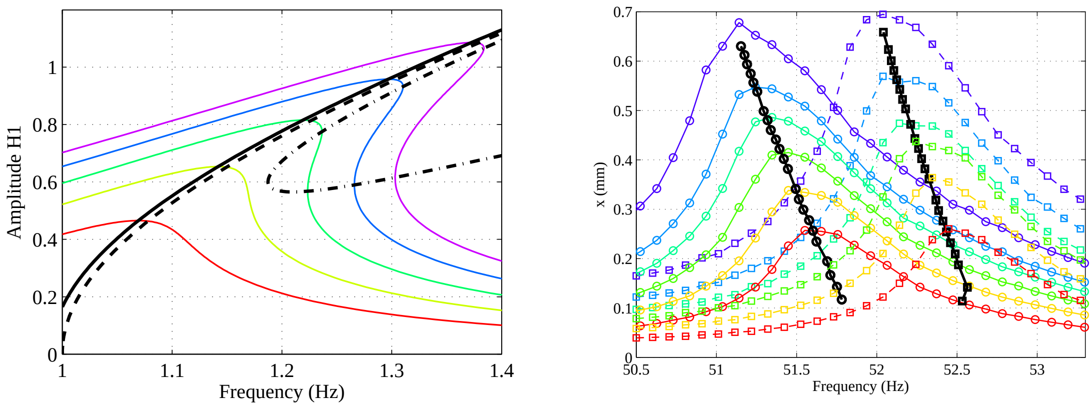


Figure 1: (left) Frequency responses (thin rainbow lines, $F$=0.1 0.15 0.2 0.25 0.3), continuation of the amplitude (full black line) and phase (dashed black line) resonances, and continuation of the fold point (dash-dotted black line) for the system (1) with $\omega_0 = 1$, $\Gamma_1 = 1$, $\Gamma_2 = 0$ and $\mu = 0.1$. (right) Frequency responses for a piezoelectric bimorph beam in open (square dashed rainbow lines) and short-(circle full rainbow lines) circuit for base accelerations 0.40, 0.60, 0.80, 1.00, 1.20, 1.56 m.s$^{-2}$, and corresponding backbone curves for open (black square) and short- (black circle) circuit).

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