Asynchronous modes of vibration in a heavy-chain model with linear and rotational springs

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Summary. Asynchronous modes of vibration in a heavy-chain model with transversal linear springs were already studied by the authors in a previous paper. Now, the effect of rotational springs that model longitudinal flexural stiffness is also taken into account, in an attempt to approximate to an even better level cylindrical shells under pre-stressing, resorting to the analogy between their behaviour and that of beams on elastic foundations. Asynchronous modes are associated to localized free oscillations. Typically, linear structural systems have synchronous modes, in which all physical coordinates vibrate in unison, that is, they reach their maximum/minimum values simultaneously and the response frequency content is unique throughout. That is not the case for asynchronous modes, because, due to the localization, a part of the system remains at rest, while other parts vibrate with modal frequency. This is the clue to understand multimodal asynchronous response, for which the response frequency content is not unique throughout. Of course, asynchronous modes are exceptional findings that require specific relationships between the system control parameters, but they may be of technological relevance in the design of vibration controllers and energy harvesters.

Introduction

Classical modes are characterized by undamped free-vibration motions in which all physical coordinates attain their maximum/minimum values simultaneously or synchronously. Yet, some systems for a special tuning of control parameters may exhibit an altogether different behaviour, that is to say, while some parts vibrate in unison, other ones stay at rest. Hence, localized free vibration is the distinguishing feature of the asynchronous modes. When initial conditions lead to a combination of an asynchronous mode with other modes, which may be synchronous or not, the frequency content of the response is clearly different in different parts of the system. Such a behaviour has already been captured in non-conservative [1,2] and conservative [3] systems alike. The present study explores still another example of the latter ones, namely the heavy-chain model that bears similarities with offshore risers, cylindrical reservoirs and cylindrical pressure vessels under pre-stressing. A particular case of the heavy-chain model dealt with here was already studied by the authors [4] in a previous paper. Asynchronous modes may be potentially useful in the design of vibration controllers and energy-harvesting devices.

The model

A three-degree-of-freedom model of a heavy chain is shown in figure 1, in which hinged-hinged rigid bars of length ℓ are interconnected and constrained by transversal elastic springs of stiffness k_t and rotational elastic springs that account for the longitudinal flexural stiffness k_ℓ. Lumped masses m are positioned at the hinges. A tension T is applied upwards at the top, whereas dead weight forces P are applied downwards at the hinges.

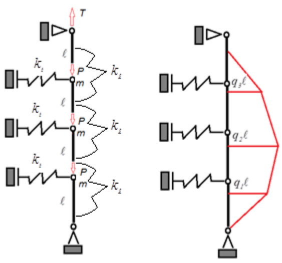

Figure 1: heavy-chain model with transversal and longitudinal springs

The linearized free-vibration equation of motion, (1) and (2), reads:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0} \tag{1}$$

$$\mathbf{K} = k_t \ell^2 \begin{bmatrix} 1+5\alpha+2\theta-5\sigma & -4\alpha-\theta+2\sigma & \alpha \\ -4\alpha-\theta+2\sigma & 1+6\alpha+2\theta-3\sigma & -4\alpha-\theta+\sigma \\ \alpha & -4\alpha-\theta+\sigma & 1+5\alpha+2\theta-\sigma \end{bmatrix}; \ \mathbf{M} = m\ell^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \ \mathbf{q} = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} \tag{2}$$

where $\alpha = \dfrac{k_\ell}{k_t \ell^2}$, $\theta = \dfrac{T}{k_t \ell}$ and $\sigma = \dfrac{P}{k_t \ell}$. The associated eigenvalue problem, with $\lambda = \dfrac{m\omega^2}{k_t}$, allows for determination of vibration modes $\mathbf{q}$ and natural frequencies ω :

$$\left(\mathbf{K} - \omega^2 \mathbf{M}\right)\mathbf{q} = k\ell^2 \begin{bmatrix} 1+5\alpha+2\theta-5\sigma-\lambda & -4\alpha-\theta+2\sigma & \alpha \\ -4\alpha-\theta+2\sigma & 1+6\alpha+2\theta-3\sigma-\lambda & 4\alpha-\theta+\sigma \\ \alpha & 4\alpha-\theta+\sigma & 1+5\alpha+2\theta-\sigma-\lambda \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} = \mathbf{0}\,. \tag{3}$$

Asynchronous modes of vibration

It can be shown that stable modes $(\lambda > 0)$ of the isolated types: (a) $\mathbf{q} = \begin{pmatrix}1 & 0 & 0\end{pmatrix}^T$, (b) $\mathbf{q} = \begin{pmatrix}0 & 1 & 0\end{pmatrix}^T$ and (c) $\mathbf{q} = \begin{pmatrix}0 & 0 & 1\end{pmatrix}^T$ are not strictly possible for $\alpha > 0$. Therefore, the non-isolated types: (d) $\mathbf{q} = \begin{pmatrix}1 & x & 0\end{pmatrix}^T$ and (e) $\mathbf{q} = \begin{pmatrix}0 & 1 & x\end{pmatrix}^T$, with $x \neq 0$, are the focus of the present paper, for some special control parameter relationships. For the sake of an illustration, figures 2 and 3 depict vibration modes of types (d) or (e), respectively, for $\alpha = 0.01$, $\sigma = 0{,}10$ and ideally tuned values for θ (indicated). Remarkably, it is seen that, in some cases, besides the perfect asynchronous modes (d) or (e), some of the synchronous companion modes may be nearly asynchronous too.

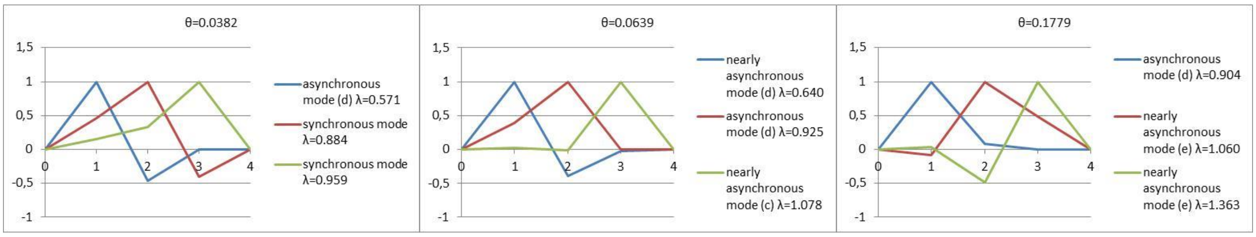

Figure 2: vibration modes under asynchronicity conditions of type (d)

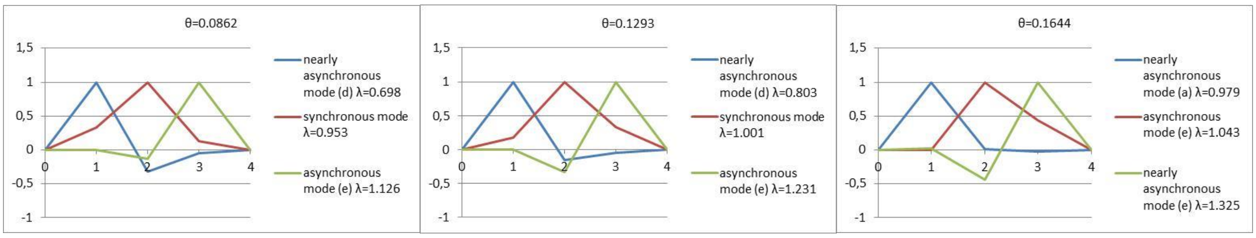

Figure 3: vibration modes under asynchronicity conditions of type (d)

Conclusion

By virtue of the classical analogy between cylindrical shells and beams on elastic foundation [5], the phenomena observed in the simple heavy-chain model can be guessed also for shell-type structures. In addition to the circumferential membrane stiffness effect that is accounted for by the transversal displacement springs, as studied in [4], the longitudinal bending stiffness effect (via the parameter α) is also considered in the present study. The bending stiffness can affect both quantitatively and qualitatively the results. Non-linear effects on asynchronicity are presently under investigation and will be timely reported.

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