Fluid-Structure Interaction Simulations of Heart Valves with Dynamic Contact

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Summary.
We decribe a fluid-structure interaction framework for simulating the behaviour of heart valves. In particular, by taking inspiration from the fictitious domain method we couple the fluid and the solid problems by means of L^2 - projections, in order to transfer data between meshes which are non-matching and randomly distributed among processors.

Introduction

In this work we present a fluid-structure interaction (FSI) framework based on the Fictitious Domain (FD) method [1] by using an L^2 - projection approach [2] for coupling the two problems and transferring data between the fluid and the solid meshes. The final goal is simulating the behaviour of the heart valve during the entire heartbeat, which requires to be able to reproduce the contact arising between the three leaflets during the closure phase. To this aim we employ a stabilized Newmark scheme and validate it by means of a FSI benchmarking configuration consisting of two beams embedded in a fluid channel.

Fluid-Structure Interaction framework: problem description

The main idea of the FD method is to model the solid phase as immersed in a background fluid phase. As it is customary, we make use of the Eulerian description for the fluid system, while we adopt the Lagrangian approach for the solid structure. The coupling between the two phases is achieved by means of the overlapping domain decomposition method in conjunction with an L^2 - projection approach, thus using a Lagrange multiplier to weakly enforce the velocity vector constraint along the interface-boundary between the solid and the fluid.
For the solution of the arising nonlinear system of discrete equations we adopt a segregated approach. More specifically, the fluid and the solid problem are solved separately: we (1) transfer the value of the structure velocity from the solid to the fluid grid, (2) solve the fluid dynamics problem by imposing the velocity constraint (3) compute and transfer the value of the Lagrange multiplier (here representing the reaction force) from the fluid mesh to the solid mesh and (4) solve the solid mechanics problem by imposing the reaction force as boundary condition. The Picard iteration method is also adopted to enforce the velocity constraint at the interface.
For coupling the two subproblems and transfer data between the two grids, a volume L^2 - projection is adopted. To this aim, we introduce the finite element spaces associated with the fluid and the solid problem, here denoted as $\mathbf{V}_f^h = \mathbf{V}_f^h(\mathcal{T}_f^h)$ and $\mathbf{V}_s^h = \mathbf{V}_s^h(\mathcal{T}_s^h)$ respectively, and the vector of Lagrange Multipliers $\boldsymbol{\lambda}_{FSI}^h$ with the related virtual variations, $\delta\boldsymbol{\lambda}_{FSI}^h \in \mathbf{M}_{FSI}^h(\mathcal{T}_s^h \cap \mathcal{T}_f^h)$, where $\mathcal{T}_s^h$ and $\mathcal{T}_f^h$ represents the solid and the fluid meshes respectively.
In the following, the projection operator $\mathbb{P} : V_s^h \rightarrow V_f^h$ is defined by focusing on the scalar case, which means that for each component of the velocity $v_{s,i}^h \in V_s^h(\mathcal{T}_s^h)$, we may find $w_{f,i}^h = \mathbb{P}(v_{s,i}^h) \in V_f^h(\mathcal{T}_f^h)$, such that the following weak-equality condition holds:

$$\int_{\mathcal{T}_s^h \cap \mathcal{T}_f^h} (v_{s,i}^h - \mathbb{P}(v_{s,i}^h))\delta\lambda_{FSI}^h \, dV = \int_{\mathcal{T}_s^h \cap \mathcal{T}_f^h} (v_{s,i}^h - w_{f,i}^h)\delta\lambda_{FSI}^h \, dV = 0 \qquad \forall \ \delta\lambda_{FSI}^h \in M_{FSI}^h \tag{1}$$

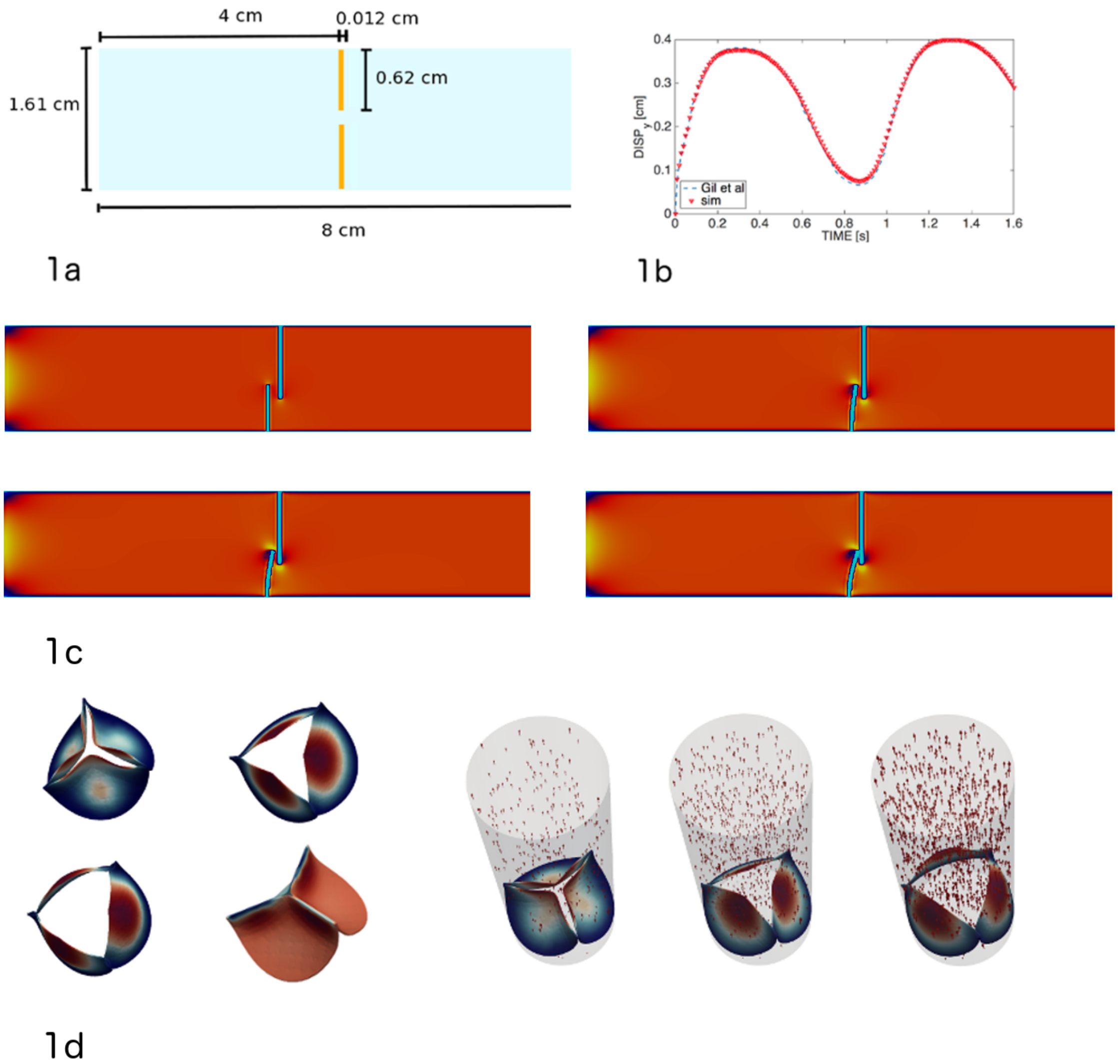
By writing v_s^h, w_f^h and $\delta\lambda_{FSI}^h$ in term of basis functions, i.e. $v_s^h = \sum_{l\in J_s} v_s^l N_s^l$, $w_f^h = \sum_{j\in J_f} v_f^j N_f^j$ and $\delta\lambda_{FSI}^h = \sum_{k\in J_{FSI}} \delta\lambda_{FSI}^k N_{FSI}^k$ (where J_s, J_f and J_{FSI} are index sets), we get the so called mortar integrals: $B_{k,l} = \int_{\mathcal{T}_s^h \cap \mathcal{T}_f^h} N_s^l N_{FSI}^k \, dV$ and $D_{k,j} = \int_{\mathcal{T}_s^h \cap \mathcal{T}_f^h} N_f^j N_{FSI}^k \, dV$. Thus, the equation (1) can be then written in the following algebraic form:

$$\boldsymbol{v}_f = \boldsymbol{D}^{-1}\boldsymbol{B}\boldsymbol{v}_s = \boldsymbol{T}\boldsymbol{v}_s \tag{2}$$

The transpose of $\boldsymbol{T}$ is used to transfer the reaction force from the fluid to the solid grid.
In the presented framework we also treat the contact conditions with the same approach, hence the L^2-projection, where instead of enforcing the weak-equality condition (1) in the volume we enforce a weak-inequality condition on the predicted contact surface [4]. The resulting system of equation is then solved by adopting a semi-smooth Newton method for the spatial discretization which allows us to treat the inequality constraints arising from the contact problem. In transient settings, for resolving the non-smooth effects caused by the non-penetration and the persistency condition, we employ a suitable stabilized Newmark scheme [3].

Numerical Results

The results are obtained within the parallel Finite Element library MOOSE (http://mooseframework.org) while the library MOONoLith (http://moonolith.inf.usi.ch) is used to identify the overlapping region between the fluid and the solid mesh



and to couple the fluid-dynamics and the solid-mechanics problems. The framework is validated by using the benchmark proposed by Gil [5] which consists of two flapping beams embedded into a two-dimensional fluid channel. The fluid is supposed to be Newtonian, while an incompressible Neo-Hookean material is used for the two solid membranes which are characterized by different stiffness. A pulsatile non reversible flow is applied as a Poiseuille flow at the inlet (on the right, see Figure 1a), whereas a non-slip condition is employed on both the bottom and the top of the fluid channel. By analyzing the displacement field of a point placed on the top beam, we get results in very good agreement with those obtained from Gil (see Figure 1b). Since the final goal is simulating the closure phase of the heart valve, we also reproduce the behaviour of two beams coming into contact during their movement as shown in Figure 1c). Finally, in Figure 1d) we show some pictures describing the displacements field in the three leaflets and the fluid flow through the heart valve.

Conclusions

In this work a framework for the FSI problem including the dynamic contact is presented. The framework is first validated by adopting the FSI benchmark proposed by Gil and then applied to simulate the behaviour of the heart valve during the heart cycle.

References

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