Axial and torsional dynamics of a distributed drill string system

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Summary. This abstract involves the dynamic (stability) analysis of distributed drill-string systems. A minimal set of parameters characterizing the linearized, axial-torsional dynamics of a distributed drill string, including delay model for the bit-rock interaction, is derived, corresponding to eight parameters for a two-sectioned drill-string (e.g., corresponding to the pipe and collar sections of a drilling system). These are used to plot the inverse gain margin of the system, parametrized in the non-dimensional parameters, effectively creating a stability map covering the full range of realistic physical parameters. This analysis reveals a complex spectrum of dynamics not evident in analysis with lumped models, thus indicating the importance of analysis using distributed models.

Introduction

The performance of rotary drilling systems used to drill boreholes in the earth is often limited by the occurrence of self-excited vibrations. Self-excited vibrations cause fatigue of drill pipes and premature failure of bits and, therefore, should be avoided. The underlying cause of the instabilities leading to such self-excited vibrations is the interaction between the drill-string dynamics and a rate-independent bit-rock interaction law through the so-called regenerative effect [3]. This effect introduces a delay based feedback in the system, as the current force on the bit, which is a function of the instantaneous depth of cut, depends on the axial position of the bit at an earlier time [2].

Distributed drill-string dynamics model

The linear axial and torsional distributed drill string dynamics are described by the dimensionless irrational transfer functions \( g_a(s), g_t(s) \), respectively. These transfer functions can be found from a transfer matrix approach [1], and relates force and torque at the bit (\( W_b, T_b \)) to axial and angular velocity at the bit (\( V_b, \Omega_b \)):

\[
\frac{V_b}{g_a(s)} = -\frac{1}{\zeta_a} g_a(s), \quad \frac{\Omega_b}{T_b} = -\frac{1}{\zeta_t} g_t(s),
\]

where \( \zeta_a, \zeta_t \) are the axial torsional pipe impedance, \( s \in \mathbb{C} \). The linearized delay-based model for the bit-rock interaction, see Fig. 1, relates weight and torque on bit to the combined depth of cut \( D(s) \) [2, 3]:

\[
W_b(s) = a\zeta\epsilon D(s), \quad T_b(s) = \frac{a^2\epsilon D(s)}{2}, \quad D(s) = \frac{N}{s} \left[ V_b(s)(1-e^{-tN\epsilon}) - R\Omega_b(s)(1-e^{-tN\epsilon}) \right],
\]

where \( a, \epsilon, \zeta, N \) are parameters dependent on bit and rock properties [3], \( R \) is the ratio of the equilibrium rate-of-penetration (ROP), denoted by \( \bar{v} \), to the equilibrium angular velocity (RPM) \( \bar{\omega} \). Finally, \( t_N = \frac{2\pi}{N\zeta} \) is the (constant) time between two successive cutters passing the same angular position at the equilibrium angular velocity \( \bar{\omega} \).

The bit-rock interaction introduces a potentially destabilizing delayed feedback into the system (as depicted in Fig. 2), which can be analyzed by employing the the Nyquist criteria on the characteristic function: \( 1 + G_a + G_t \), obtained by combining (1) and (2), where:

\[
G_a(s) = g_a(s)\frac{K_a}{s}(1-e^{-tN\epsilon}), \quad G_t(s) = g_t(s)\frac{K_t}{s}(1-e^{-tN\epsilon}), \quad K_a = \frac{a\zeta\epsilon N}{\zeta_a}, \quad K_t = \frac{Ra^2\epsilon N}{2\zeta_t}.
\]

Figure 1: Downhole bit-rock interaction, from [2, 3].

Figure 2: Block diagram of the system.
By introducing the non-dimensional time $\bar{t} = t/t_s$, where $t_s = L/c_t$ is the wave travel time for the torsional wave velocity $c_t$, we find that the complete two-section drill-string dynamics, with the characteristic equation $1 + G_a + G_t = 0$, can be specified by eight dimensionless parameters: $\Omega = \frac{\bar{N} \bar{\omega}}{2\pi}$ - Dimensionless top-drive RPM relative to the drill string travel time, $\bar{K}_a = K_a t_s$, $\bar{K}_t = K_t t_s$ - Nominal axial and torsional loop gain coefficients, $\eta_a, \eta_t$ - Pseudo reflection coefficients, given by the amount of damping of the axial and torsional dynamics, $\zeta_a = \frac{c_a}{\bar{K}_a}$, $\zeta_t = \frac{c_t}{\bar{K}_t}$: The relative size of collar to pipe cross sectional area and polar moment of inertia, respectively, and $t_p^* = t_p^* / t_s \in [0, 1]$: drill pipe (topmost section) travel time relative to total drill string travel time.

**Stability analysis results**

The derived characteristic parameters can be used to parametrize the stability properties of the system. To this end, we define the normalized inverse of the system gain margin, $M_{G,i,i} = \bigcup \{i, i \} \in \{a, t\}$, as

$$ M_{G,i} \equiv \max_{\omega \in \Omega^{180}} \left| \frac{G_i(j\omega)}{K_i} \right|, \quad \text{with} \quad \Omega^{180} = \{\omega \in \mathbb{R} : \angle G_i(j\omega) = 180 \text{ deg} \}. \quad (4) $$

This means that $\bar{K}_i M_{G,i} > 1$ (i.e., exceeds 0 dB) indicates instability. This analysis reveals complex dynamics for the distributed system, see Fig 3, not evident in analysis obtained using lumped models. The main trends that emerge are that decreases in the inverse gain margin (leading to a more stable system) is obtained by:

1. Decreasing the reflection coefficient, c.f. left and right side of Fig. 3.
2. Increasing the angular top-drive velocity, $\Omega$. The magnitude of the peaks can be seen to decrease for increasing $\Omega$ in Fig. 3.

The presented result also enables a more structured approach to analyzing the non-local (non-linear) dynamics through the derivation of the minimal set of characteristic parameters.

**References**


**Figure 3:** 3D plot of the inverse gain margin, $M_{G,a}$, for a two section pipe, for large amount of damping ($\eta_a = 0.4$, left) and ($\eta_a = 0.8$, right). The stability lobes, characteristic of delay systems, can be clearly seen for $\Omega_N < 1$, while for $\Omega_N > 1$ the more complex interaction between (anti-) resonances of axial dynamics in $g_a(s)$ and the delay term creates a series of intersecting ridges and troughs, the most pronounced of which is, indicated by red lines.