Binary gas mixture in a high speed channel

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The viscous, compressible flow in a 2D wall-bounded channel, with bottom wall moving in positive x direction, simulated using the direct simulation Monte Carlo (DSMC) method *[1, 2, 3, 4, 5, 6, 7, 8, 9, 10]*, has been used as a test bed for examining different aspects of flow phenomenon and separation performance of a binary gas mixture at Mach number $Ma = U_w / \sqrt{(\gamma \; k_B \; T_w / m)}$ in the range $0.1 < Ma < 30$, and Knudsen number $Kn = (1/(\sqrt{2} \pi d^2 n_d \; H)$ in the range $0.1 < Kn < 10$. Here, H is the channel width, U_w is the wall velocity, T_w is the wall temperature, m, and d are the molecular mass and molecular diameter, n_d is the number density, and k_B is the Boltzmann constant. The generalized analytical model is formulated which includes the fifth order differential equation for the boundary layer at the channel wall in terms of master potential (χ), which is derived from the equations of motion in a rectangular $(x-y)$ *plane*. The starting point of the analytical model is the Navier-Stokes, mass, momentum and energy conservation equations in the rectangular $(x-y)$ coordinate, where x and y are the streamwise and wall-normal coordinates. The linearization approximation is used, where the equations of motion are truncated at linear order in the velocity and pressure disturbances to the base flow, which is an isothermal compressible Couette flow. Additional assumptions in the analytical model include high aspect ratio (length of the channel L is large compared to the height H), constant temperature in the base state (isothermal condition), and low Reynolds number (laminar flow). In this limit, the gas flow is restricted to a boundary layer of thickness $(Re^{-1/2} H)$ at the wall of the channel. The solutions of the generalized analytical model in a high-speed channel are compared with direct simulation Monte Carlo (DSMC) simulations. The comparison reveals that the boundary conditions in the simulations and analysis have to be compared with care. The commonly used 'diffuse reflection' boundary conditions at solid walls in DSMC simulations result in a non-zero slip velocity as well as a 'temperature slip' (gas temperature at the wall is different from wall temperature). These have to be incorporated in the analysis in order to make quantitative predictions. When these precautions are taken,there is excellent agreement between analysis and simulations, to within 10%, for Mach number Ma in the range $0.1 - 30$, and Knudsen number Kn in the range $0.1 - 10$.

Key words: High speed channel flow, DSMC Simulation, rarefied gas flow.

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