Experiments and Analysis on Nonlinear Vibrations of a Post-buckled Stepped Beam

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Summary. Experimental and analytical results are presented on nonlinear and chaotic vibrations of a stepped beam constrained by an axial elastic spring. The rectangular cross section of the beam is changed to H shape at the mid span of the beam. One end of the beam is clamped and the other is simply-supported. The beam is compressed to the post-buckled state by the spring in the axial direction. In the experiment, the beam is excited laterally under periodic acceleration, and the dynamic responses of the beam are measured. In the analysis, the beam is divided into a few segments. The deflection of the beam is expanded with the mode shape function that is expressed with the product of truncated power series and trigonometric functions. Taking the axial displacement, the deflections, slopes, bending moments and shearing forces at the nodes of the segments as unknown variables, nonlinear coupled ordinary differential equations are derived with the Galerkin procedure. Neglecting the axial inertia of the beam, the axial displacements at the nodes are expressed as nonlinear functions of the deflections, slopes, bending moments and shearing forces, nonlinear responses are calculated with the harmonic balance method and with the direct time integration. Fairly good agreements are obtained between results of experiment and analysis.

Introduction

Recently, technology of a micro electro-mechanical system (MEMS) has been developed drastically. Micro devices such as an acceleration pickup and an optical scanner are widely utilized. These devices are composed with elements of thin elastic structures. The elements have complicated shape with discontinuous cross section like a stepped beam or combined configuration of beam and plate. When the thin beams are subjected to periodic force and large amplitude resonance are generated, nonlinear responses are easily generated. Therefore, in this paper, both experimental and analytical results are presented on nonlinear and chaotic vibrations of a stepped beam.

Procedure of Experiment

Fig.1 shows the stepped beam and its fixture. A thin phosphor bronze beam with thickness h=0.30 mm, breadth b=40 mm and length L=140 mm is clamped at one end and simply-supported at the other end. Four thin phosphor bronze beam (thickness 0.31 mm, breadth 4.9 mm, length 34 mm) are attached to the mid span of the beam, then the cross section is locally changed to H-shaped. At the simply-supported end, the beam is connected to an elastic plate by the strips of adhesive films. The elastic plate is clamped by the slide block and works as the axial spring. The beam is compressed by the axial spring, then the beam is deformed to the post-buckled configuration. To find fundamental properties of the beam, the linear natural frequencies and the restoring force are inspected. The post-buckled beam is excited laterally with an electromagnetic exciter. The beam is subjected to gravitational acceleration and periodic acceleration $a_d\cos 2pf\,t$, where f is the excitation frequency and a_d is the peak amplitude of acceleration. The dynamic responses of the beam are measured under three magnitudes of axial compression. In typical condition, chaotic responses are observed. The responses are inspected with the frequency response curves, the Fourier spectra, the Poincaré projections and the maximum Lyapunov exponents.

Procedure of Analysis

In the analysis, the beam is divided into three segments, two of which corresponds to the parts with original rectangular cross section, the other corresponds to the part with H-shaped cross section. A vector $\{w_{en}\}$ that consists of nodal deflection w_n, slope s_{xn}, bending moment m_{xn} and shearing force q_{xn} at the both nodes of the n-th segment is introduced, then the deflection w_n in the n-th segment is expressed with the coordinate function $\{\zeta_n\}$, following the similar manner of the finite element procedure.

$$w_n(\xi_n,\tau)=\sum_{j=1}^{8}w_{enj}(\tau)\zeta_{nj}(\xi_n),\{\zeta_n\}=\{\bar{Z}_n\}^T([D_n][Z_n])^{-1},$$
$$\bar{Z}_{ni}=\sum_{l=1}^{2}\sum_{k=1}^{4}\delta_{i,f(k,l)}\left\{(2\xi_n)^{k-1}\cos(l-1)\pi(\xi_n+1/2)\right\},\ f(k,l)=4(l-1)+k \quad (1)$$

In the above equations, $\{\bar{Z}_n\}$ is a vector composed of the mode shape function Z_{ni} that is the product of truncated power series and trigonometric functions, $[Z_n]$ is a 8×8 matrix consists of Z_{ni} and its first, second and third order derivatives, $[D_n]$ is a 8×8 matrix consists of parameters of the n-th segment. Introducing the global nodal vector $\{\hat{b}\}$ which includes the nodal vector $\{w_{en}\}$ of the all segments, and the vector $\{\hat{d}\}$ which consists of axial displacement of all nodes, and applying the Galerkin procedure, the nonlinear governing equation of the beam is reduced to a set of ordinary differential equations as follows.

$$\sum_q \hat{B}_{pq}\hat{b}_{q,\tau\tau} + \sum_q \hat{C}_{pq}\hat{b}_q + \sum_q\sum_v \hat{D}_{pqv}\hat{b}_q\hat{d}_v + \sum_v\sum_q \hat{D}_{pvq}\hat{d}_v\hat{b}_q + \sum_q\sum_r\sum_s \hat{E}_{pqrs}\hat{b}_q\hat{b}_r\hat{b}_s - \hat{F}_p - \hat{G}_p\left(p_s + p_d\cos\omega\tau\right) = 0 \tag{2}$$

$$\sum_v \hat{C}_{tv}\hat{d}_v + \sum_r\sum_s \hat{D}_{trs}\hat{b}_r\hat{b}_s - \hat{F}_t = 0 \qquad p,q,r,s = 1,2,\cdots,4(N+1),t,v = 4(N+1)+1,4(N+1)+2,\cdots,5(N+1) \tag{3}$$

Solving $\{\hat{d}\}$ in terms of $\{\hat{b}\}$ in Eq.(3), and then substituting it to Eq.(2), the axial displacements can be removed in the reduced governing equation. Neglecting the time variant terms, static deflection due to the static lateral acceleration and the axial initial displacements is obtained. Next, the ordinary differential equation is transformed to the equation in terms of the dynamic variable $\tilde{b}_j$ which is measured from the static equilibrium position. Furthermore, the ordinary differential equations are transformed to the standard form in terms of normal coordinates b_i corresponding to the linear natural modes of vibration $\tilde{\xi}_j$ at the static equilibrium position of the beam. Dynamic responses can be calculated with the harmonic balance method and the numerical integration.

Results and Discussion

Equivalent moment of cross section of the H-shaped part and the initail deflections are indentified by comparing the experimental and analytical results of the post-buckled deformation (Fig.2) and characteristics of restoring force under an concentrated lateral force on the beam (Fig.3), for three conditons of the magnitude of axial compressive force. Fig.4 shows the nonlinear frequency curves of the beam comparing the analytical and experimental nonlinear responses. In the figure, the black and gray curves are the stable and unstable periodic responses, respectively, calculated by the harmonic balance methos. The principal resonance (1:1) and the sub-harmonic resonance (1:2) of the order 1/2 of the lowest mode appeards corresponding to the softening-and-hardening characteristics of the restoring force. The results of direct numerical integration, shown with the blue curves, almost follows the stable periodic responses. At the non-dimensional frequency ω=12.92, chaotic response are obtained in the analysis. The red curves in the figure presents the experimental resluts. Fiary good agreements are obtained between experimental and analytical periodic responses. Chaotic responses are also observed near the frequency ω=12.5. Fig.5 shows the experimental and analytical results of the Poincaré projection of the chaotic responses in the phase space of deflection and velocity. Both experimental and analytical results show similar fractal patterns.

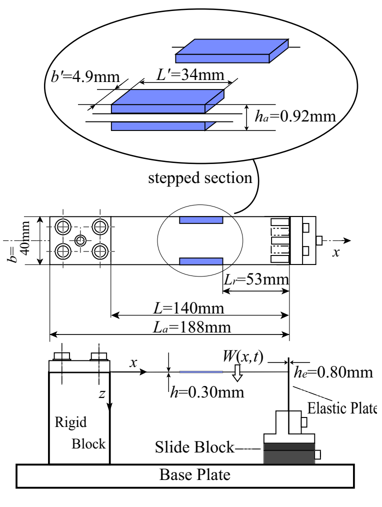


Fig.1 Beam and fixture

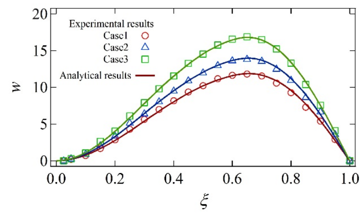


Fig.2 Post-buckled deformation

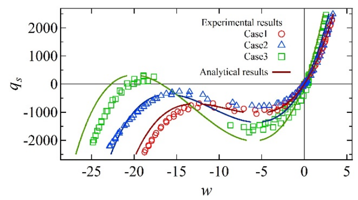


Fig.3 Characteristics of restoring force

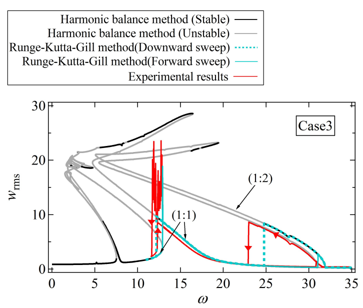
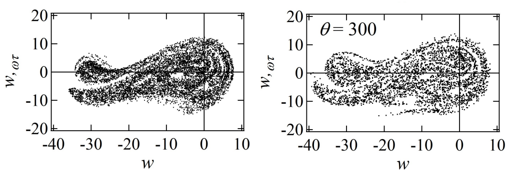


Fig.4 Nonlinear frequency response curves



Experiment Analysis

Fig.5 Poincaré projection of chaotic responses

References

[1] Nagai K., Maruyama S., Sakaimoto K. and Yamaguchi T., (2007) Experiment on Chaotic Vibrations of a Post-buckled Beam with an Axial Elastic Constraint. *J. Sound and Vibration* **304**:541-555.