Influence of frictional mechanism on chatter vibrations in cutting process

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Summary. An analysis of a nonlinear one degree of freedom model of cutting process is presented in the paper. Classical regenerative mechanism of chatter is enriched in an additional friction phenomenon which generates frictional chatter. Additionally, nonlinear cubic stiffness of a tool is taken into account. A goal of the paper is to detect a mutual interaction between regeneration and frictional effect. The model is solved by means of the multiple time scale method. Stability of cutting process is checked in order to determine stability lobes diagrams and to find an influence of friction on the process. Finally, the maps of chatter amplitudes are presented and new frictional stability lobes diagrams are proposed to analyse an influence of friction.

Introduction

Nowadays, cutting process is still one of the most popular manufacturing method. The efficiency of a machining operation is determined by the metal removal rates, cycle time, machine down time and tool wear. A phenomenon called regenerative chatter is a primary factor that limits process efficiency in machining. However, much later another chatter mechanism produced by friction has been developed and called frictional chatter. The regenerative effect is related to the wavy workpiece surface generated by the previous cutting tooth pass [3]. While, the frictional mechanism results from friction force occurring between the tool and the workpiece. Although, trace regeneration and friction are the most important in practical operations there are little papers which consider regenerative and frictional mechanisms together. Friction always exists in real cutting process therefore, excluding this phenomenon is rather a big simplification. Generally, chatter is a dynamic instability that can limit material removal rates, cause a poor surface finish and even damage the tool or the workpiece [4].
In order to get knowledge about an influence of frictional chatter on regenerative one and complete an mathematical approach, the mathematical model of cutting is developed and solved with the help of the method of the multiple time scales [2]. An explanation of mutual interaction between frictional and regenerative mechanisms is the main purpose of the paper. Dynamics of one degree of freedom model of a cutting process is studied with a special attention devoted to stability problems of trivial and non-trivial solutions.

Mathematical model

To analyse regenerative and frictional mechanism of chatter, one degree of freedom model of orthogonal cutting is used. In order to explain an interaction between regenerative and frictional mechanisms only the feed direction (*x*) is considered here. From our point of view the feed direction is more important, specially because the regenerative mechanism depends on a tool position in *x* (feed) direction and friction between a tool and a chip. The tool is modelled as lumped mass with a nonlinear spring and a linear damper. The nonlinear spring is sometimes used in literature to model nonlinear properties of the tool and tool holder, although a linear spring is also accepted. The equation of tool motion is presented as

$$x''(t) + \delta x'(t) + \omega_0^2 x(t) + \gamma x(t)^3 - \alpha\left[h_0 - x(t) + x(t-\tau)\right] - \beta\left[\operatorname{sgn}(v_r) - a_r v_r + b_r v_r^3\right] = 0 \tag{1}$$

where, δ is damping, γ nonlinear stiffness coefficient, ω_0 natural frequency, α corresponds to cutting resistance of regenerative force, while β is the cutting resistance of the friction force component. The regenerative force depends on the depth of cut h_0, the present tool position $x(t)$ and the previous position $x(t-\tau)$. Where the time delay τ is connected with spindle speed Ω. v_r means relative velocity between the tool and the workpiece

$$v_r = v_c - x'(t) \tag{2}$$

Analytical solution

Non-dimensional equation of motion of the cutting tool (1) is solved analytically with the help of the multiple time scale method [1]. At the beginning it is assumed that the sign of relative velocity v_r is always positive, thus sign function equals 1. Next, the fast T_0 and the slow T_1 time scales are introduced and defined as follows

$$T_0 = t, T_1 = \varepsilon t \tag{3}$$

where ε is a formal small parameter.
Then a solution in the first order approximation has the form:

$$\begin{gathered} x(t) = x_0(T_0, T_1) + \varepsilon x_1(T_0, T_1) \\ x(t-\tau) = x_\tau = x_{0\tau}(T_0, T_1) + \varepsilon x_{1\tau}(T_0, T_1) \end{gathered} \tag{4}$$

It is assumed that:

$$\omega_0^2 = \omega^2 + \varepsilon\sigma, \delta = \varepsilon\hat{\delta}, \gamma = \varepsilon\hat{\gamma}, \alpha = \varepsilon\hat{\alpha}, \beta = \varepsilon\hat{\beta} \tag{5}$$

where: ε is a formal small parameter. Next, in order to facilitate notation, the tilde is omitted. After substitution into Eq.(1) and applying the method results in the modulation equations in the form

$$
\begin{aligned}
a'(T_1) &= -\frac{1}{2}\delta a(T_1) - \frac{1}{2}\alpha a(T_1)\sin\tau + \frac{1}{2}\beta a_r a(T_1) - \frac{3}{8}\beta b_r a(T_1)^3 - \frac{3}{2}\beta b_r v_c^2 a(T_1) \\
\beta'(T_1) &= \frac{1}{2}\mu\alpha + \frac{1}{2}\sigma + \frac{3}{8}\gamma a(T_1)^2 - \frac{1}{2}\alpha\cos\tau
\end{aligned}
\tag{6}
$$

Solving Eqs.6, one trivial (a_1) and two non-trivial periodic solutions ($a_{2,3}$) are found:

$$
\begin{aligned}
a_1 &= 0 \\
a_{2,3} &= 2\sqrt{\frac{a_r - \delta/\beta}{3b_r} \mp \frac{\alpha\sin\tau}{3\beta b_r} - \frac{d^2}{\tau^2}}
\end{aligned}
\tag{7}
$$

The stability of the trivial solution a_1 is important from practical point of view because if this solution is stable, the cutting process is stable and chatter vibrations do not occur. While the trivial solution is unstable, chatter appears. Therefore, stability of the solution is a big importance. The first nontrivial solution a_2 (corresponding to periodic orbit in the original system) is stable exactly when the trivial solutions is unstable, but the second nontrivial solution a_3 is stable in the regions where the trivial solutions is stable. Thus, two solutions: trivial a_1 and nontrivial a_3 may exist in the same region of SLD depending on initial conditions. Both, the region of existence of nontrivial solutions (a_2 and a_3) is presented in Fig.1a where an influence of α is observed. Interestingly, that in the first order approximation, chatter vibrations do not depend on cubic nonlinearity determined by coefficient γ. Similar diagrams with unstable lobes are obtained on the plane $\Omega - \beta$ (Fig.1b).

a) b)

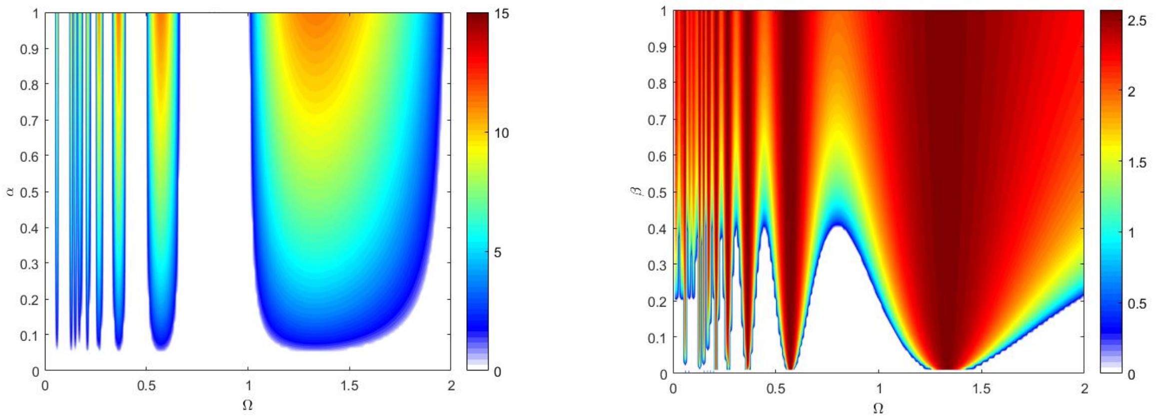

Fig1. Influence of vibration regeneration mechanism expressed by α (β=0.1) a), and friction effect expressed by β (α=0.1) on stability of non-trivial solution (the colors indicate the value of vibrations amplitude).

Conclusions

Chatter vibrations as a result of classical regenerative and added frictional mechanisms are discussed here in order to investigate a mutual interaction between them. An analytical method of the multiple time scales is used successfully for solving of the nonlinear problem of cutting process. Classical unstability lobes generated by the regenerative effect are modified by the action of friction. The friction phenomenon widens unstable cutting regions, but on the other hand reduces chatter vibrations amplitude. The regenerative model of cutting with friction has trivial and two nontrivial (corresponding to periodic) solutions. The nontrivial and trivial solutions can exists simultaneously at specific cutting speeds because both solutions can be stable. From a practical point of view it means that any disturbance causing a change of initial conditions can lead to chatter even in the region where the classical regenerative cutting process is expected to be stable. A change of initial conditions can be caused for example by chip break. The interesting phenomenon of reverse unstable lobes (not presented here) can be shown on the plane of rotational speed - friction force coefficient. Untypical behaviour is manifested by bigger vibrations amplitude for small β value than for big one.

References

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