Towards the Adoption of the Stiffness Evaluation Procedure as Non-intrusive, Non-linear Model Reduction Method in Car Crash Simulations

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Summary. Making crash test simulations faster is a great challenge due to their highly non-linear behavior and closed-source simulation software. We adopt the non-linear, non-intrusive model reduction method Stiffness Evaluation Procedure (STEP) to crash test simulations. Since STEP does not scale well and limits the dependence of the non-linear term on parameters, we extend it to overcome these disadvantages by applying a local bases approach and assuming a more general Taylor extension.

Setting

Motivation

For the evaluation of the crashworthiness during the design phase of a modern car, virtual crash tests are simulated with crash software rather than performed with expensive prototypes. Though all car manufactures use big computer clusters for their simulations, the response time, i.e., the time after the engineer has access to results for subsequent design decisions, varies still between one day and one week[1]. Thus, the industry is in need of faster but still reliable simulation tools. One promising approach can be model order reduction, which substitutes the underlying differential equations by equations with much smaller dimensions but tries to keep the most important system behavior.

Identification of Non-linear Behavior

While so-called linear model reduction designed for linear systems is considered as mature and production ready, a crash simulation behaves highly non-linearly due to large deformations, non-linear material laws, and contacts. This demands non-linear model reduction, i.e., reduction methods designed for non-linear systems. Since not all parts of the car model exhibit non-linear dynamics, a separate treatment seems to be sensible—parts with linear behavior should be reduced with linear reduction methods and parts with non-linear behavior with non-linear methods or not reduced at all. This separation can be identified with modern clustering techniques adopted for crash scenarios, see [1].

Non-intrusive Methods

There exist a lot of promising non-linear model reduction techniques like POD-DEIM and its variations, Hyper-reduction with local bases based on GNAT, ECSW, or PGD to name only a few. All listed methods need access to the underlying differential equation. Unfortunately, this access is not possible in closed-source simulation software, which is used by the car industry. Since our research should benefit not only the academic sector, we assume a black box setting, i.e. no or only little direct access to the differential equations of the system at hand is possible. This leads to so-called non-intrusive model reduction methods.

Stiffness Evaluation Procedure

The Stiffness Evaluation Procedure originally published in [2] is a non-intrusive method that focuses on approximating the unknown, non-linear stiffness force vector by probing the original model with static analysis in order to find a Taylor approximation for the original non-linear stiffness force vector.
A non-linear, mechanical system of the form

$$\boldsymbol{M} \cdot \ddot{\boldsymbol{q}} + \boldsymbol{D} \cdot \dot{\boldsymbol{q}} + \boldsymbol{K} \cdot \boldsymbol{q} + \boldsymbol{\Gamma}(\boldsymbol{q}) = \boldsymbol{f}^{\text{ext}}$$

with the coordinates of displacement $\boldsymbol{q} \in \mathbb{R}^N$ coming from an FE discretization in space is assumed. The non-linear stiffness force vector $\boldsymbol{f}^{\text{int}}$, which is present in the equations of motion of a car crash, is already separated in a linear part $\boldsymbol{K} \cdot \boldsymbol{q}$ and purely non-linear part $\boldsymbol{\Gamma}(\boldsymbol{q})$ in the sense that it does not contain a first-order polynomial in its Taylor approximation. $\boldsymbol{\Gamma}$ is not known a priori but $\boldsymbol{K}$ can be exported by almost every FE software as the tangential stiffness matrix used in the Newton-Raphson iterations of implicit solvers.
In the first step, the full system is reduced with a linear method of choice resulting in the system

$$\overline{\boldsymbol{M}} \cdot \ddot{\overline{\boldsymbol{q}}} + \overline{\boldsymbol{D}} \cdot \dot{\overline{\boldsymbol{q}}} + \overline{\boldsymbol{K}} \cdot \overline{\boldsymbol{q}} + \overline{\boldsymbol{\Gamma}}(\overline{\boldsymbol{q}}) = \overline{\boldsymbol{f}}^{\text{ext}} \tag{1}$$

with reduced coordinates $\overline{\boldsymbol{q}}$ of dimension $n \ll N$ and still unknown $\overline{\boldsymbol{\Gamma}}$.
The main idea of STEP consists of approximating $\overline{\boldsymbol{\Gamma}}$ by a Taylor sum up to order three:

$$\overline{\Gamma}_i(\overline{\boldsymbol{q}}) \approx \sum_{j=1}^{n} \sum_{k=1}^{n} a_{ijk} \overline{q}_j \overline{q}_k + \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} b_{ijkl} \overline{q}_j \overline{q}_k \overline{q}_l \tag{2}$$

[1] Source: Personal communication with engineers from Audi AG and Daimler AG.

The unknown coefficients a_{ijk} and b_{ijkl} can be found by choosing $\overline{\boldsymbol{q}}$ in a smart way and solving (1) for $\overline{\boldsymbol{f}}^{\text{ext}}$ with the closed-source simulation software. It is described in [2] how to best choose $\overline{\boldsymbol{q}}$ in order to reduce the amount of static analysis needed to be able to solve equation (1). The static analysis itself is invoked automatically by optimized scripts.
In the current setup, i.e., $\overline{\boldsymbol{\Gamma}}$ being a Taylor approximation up to order three, STEP scales offline with $\mathcal{O}(n^3)$ in the sense that n^3+n^2 static analyses need to be performed by the FE software to calculate the same amount of coefficients. But also the online cost, i.e., the simulation time of the reduced model increased significantly comparing to a reduced linear model. For a linear model, the reduced equation of motion, which is evaluated by a transient solver many times, only consists of matrix vector multiplications, which scales with $\mathcal{O}(n^2)$. However, the evaluation of the non-linear approximation (2) scales with $\mathcal{O}(n^4)$ for the non-linear model reduced with STEP.
It can be advantageous to define $\overline{\boldsymbol{\Gamma}}$ locally on k sets of (almost) independent coordinates of $\mathbb{R}^n$, thus reducing the cost to $k^{-2}\mathcal{O}(n^3)$. The curse of dimensionality can also be beaten with other ansatz functions for $\overline{\boldsymbol{\Gamma}}$ like a kernel expansion. Since a kernel expansion is the linear combination of a fixed kernel function, e.g., a Gaussian kernel evaluated at center points, its evaluation scales with the number of center points. The coefficients of the kernel expansion can be determined with a Support Vector Machine known from statistical learning.
STEP in its original form only allows $\boldsymbol{\Gamma}$ to depend on $\boldsymbol{q}$. This does not cover the full spectrum of a crash simulation since $\boldsymbol{f}^{\text{int}}$ can also depend on other parameters $\boldsymbol{\mu} \in \mathbb{R}^m$. Thus, we extend STEP by adding these additional parameters to the Taylor approximation (2) of $\overline{\boldsymbol{\Gamma}}$ which allows for a parametric model reduction. The new Taylor approximation for $\overline{\boldsymbol{\Gamma}}(\boldsymbol{p})$ with the combined vector $\boldsymbol{p} = (\overline{q}_1, \ldots, \overline{q}_n, \mu_1, \ldots, \mu_m) \in \mathbb{R}^{n+m}$ is

$$\overline{\Gamma}_i(\boldsymbol{p}) \approx \sum_{j=1}^{n+m} \sum_{k=1}^{n+m} a_{ijk}\overline{p}_j\overline{p}_k + \sum_{j=1}^{n+m} \sum_{k=1}^{n+m} \sum_{l=1}^{n+m} b_{ijkl}\overline{p}_j\overline{p}_k\overline{p}_l \tag{3}$$

with new coefficients a_{ijk} and b_{ijkl}.

Implementation

We evaluate the reduction quality and speed of STEP with the publicly available 2001 Ford Taurus model by the National Crash Analysis Center simulated with LS-DYNA by the Livermore Software Technology Corporation. Since STEP without our modifications scales with $\mathcal{O}(n^3)$, we focus on the analysis of individual components instead of the complete car.
The linear model reduction is performed by the software package MatMorembs. In a first step, Matlab is used to solve the reduced equation (1) after determining the coefficients by automatically starting several static analyses in LS-DYNA and the automated extraction of the results, cf. Fig. 1. Later, the reduced order model can be implemented directly in LS-DYNA.

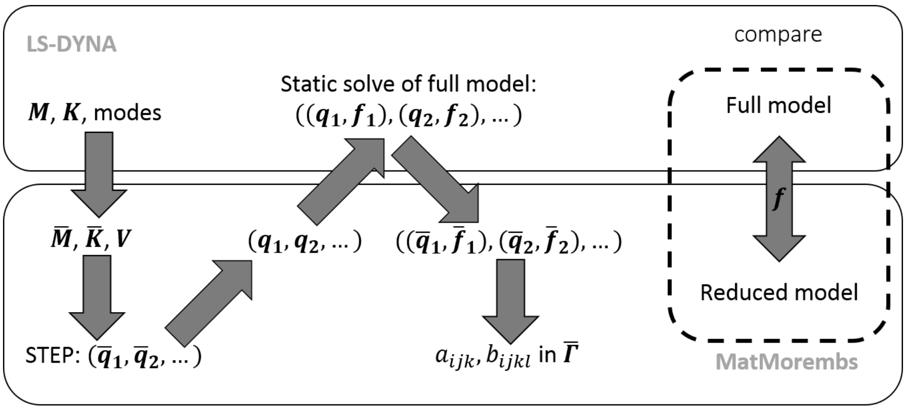

Figure 1: Workflow of STEP mostly implemented in Matlab.

References

[1] Grunert, D., Fehr, J. (2016) Identification of Nonlinear Behavior with Clustering Techniques in Car Crash Simulations for Better Model Reduction. *Advanced Modeling and Simulation in Engineering Sciences* **3(1)**:1–19.

[2] Muravyov, A. A., Rizzi, S. A. (2003) Determination of Nonlinear Stiffness with Application to Random Vibration of Geometrically Nonlinear Structures. *Computers & Structures* **81(15)**:1513–1523.