Parametric vibrations of a rotating thin-walled composite blade subjected to base excitation

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Summary. In the presented paper dynamics of a rotating flexible composite thin-walled box-beam subjected to additional base excitation (support movement) is investigated. The assumed blade spacial orientation and laminate centrifugal asymmetric stiffness (CAS) material configuration results in twist/in-plane bending/in-plane shear deformation coupling. The equations of motion of the structure are derived using the Hamilton's least action principle. Resulting partial differential equations are discretised using the Galerkin's method and then transformed to dimensionless form. It is found two of the coefficients of this governing equation are time-varying ones and depend on the system angular velocity and the base excitation frequency. This results in parametric excitation and external forcing terms correspondingly. The multiple time scale method is used to determine the boundaries of dynamic stability regions. Studies regarding selected structural parameters like reinforcing fibers orientation angle and hub radius to beam length ratio are presented.

Introduction

Rotating, flexible thin-walled beam structures made of advanced anisotropic composite materials are widely used in aerospace, automobile, robotic and civil industries due to their outstanding mechanical and physical properties, such as high strength/stiffness to weight ratios, good fatigue and corrosion resistance characteristics.

Very often these structures are operated as on-board machines – examples are automotive turbocharger, aircraft turbines, helicopter rotor or wind turbines subjected to gravity and even magnified by the additional earthquake shocks [1, 3, 5]. In all these cases the base excitation affects the overall performance of the rotating structure. This is exhibited by the increased lateral blade vibrations and in limit states it may lead to the large responses and the dynamic instability phenomenon.

The problem of a rotating and moving beam has been discussed in the literature, but most of research deals with slender specimens made of isotropic material [6, 8]. Very limited number of studies is devoted to rotor structures made of composite materials despite the directional properties provided by fiber reinforced composite materials that can be used to enhance the system response characteristics. Therefore a proper mathematical model and further comprehensive studies on the composite beam rotor considering multi-source excitations are of prime scientific and engineering interest.

Problem formulation

The system under consideration comprises a slender, straight and elastic composite thin-walled beam clamped at the rigid hub that is experiencing rotational motion as well as to-and-fro motion – see Figure 1. The temporary position of the hub is given by a position vector $\xi(t)$ as well as a rotation angle $\psi(t)$. For simplicity, the direction of the hub angular velocity $\omega = \dot{\psi}(t)$ is constant and fixed in space (parallel to inertial Z_S axis at all times) and the hub is assumed to be moving along ($X_S = X_0$) axis only. It is assumed that the above mentioned translational motion of the centre of the hub is given by a periodic oscillation $\xi(t) = \xi_0 \sin \upsilon t$, where denotations ξ_0 and υ correspond to the amplitude and the frequency of to-and-fro motion respectively. Detailed information on the assumptions made in the mathematical model of the system and further adopted simplifications are given in authors previous papers [2, 4].

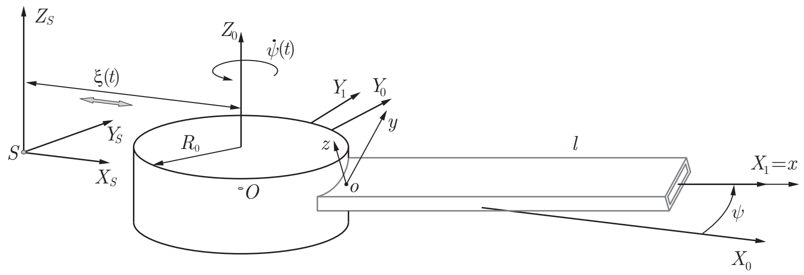

Figure 1: Schematic diagram of a rotating beam with in-plane base movement.

The equations of motion and boundary conditions of the rotating beam are derived according to the extended Hamilton's principle of the least action:

$$\delta J = \delta \int_{t_1}^{t_2} (T - U)\mathrm{d}t = 0, \tag{1}$$

where J is the action, T is the kinetic energy, U is the potential energy. Step by step derivation of the full system of equations of motion for a complete model including both transversal/lateral bending directions, shear deformations and

warping effects, as well as arbitrary pre-setting angle and non-constant rotational speed can be found in authors papers [2, 4]. In the presented case the co called Circumferentially Asymmetric Stiffness (CAS) lamination scheme is assumed. This simplifies the general system of governing equations and results in flap-wise bending/twist modes coupling.
The governing equations are expressed in terms of unknown kinematic variables (displacements) of its cross-section reference point located at (ox) axis as follows:

- w_0 lead-lag plane displacement

$$b_1\ddot{w}_o - 2b_1\dot{u}_o\,\omega - b_1 w_o\omega^2 - b_1\xi_0\nu^2\sin\nu t\sin\omega t - a_{55}\vartheta_y' - a_{55}w_o'' - (T_x w_o')' = 0 \tag{2}$$

with boundary conditions $\quad w_o\big|_{x=0} = 0, \qquad \left(\vartheta_y + w_o'\right)\big|_{x=l} = 0;$

- ϑ_y in plane transverse shear angle

$$B_4\ddot{\vartheta}_y - B_4\omega^2\vartheta_y + a_{55}(\vartheta_y + w_o') - a_{33}\vartheta_y'' - a_{37}\varphi'' = 0 \tag{3}$$

with boundary conditions $\quad \vartheta_y\big|_{x=0} = 0, \qquad \left(a_{33}\vartheta_y' + a_{37}\varphi'\right)\big|_{x=l} = 0;$

- φ twist angle

$$(B_4 + B_5)\ddot{\varphi} + (B_4 - B_5)\omega^2\varphi - a_{37}\vartheta_y'' - a_{77}\varphi'' - (T_r\varphi')' = 0 \tag{4}$$

with boundary conditions $\quad \varphi\big|_{x=0} = 0, \qquad \left(a_{37}\vartheta_y' + a_{77}\varphi'\right)\big|_{x=l} = 0$

In all foregoing relations B_i and b_i factors depict the inertia terms and a_{ij} ones correspond to beam stiffnesses [2]. Term $T_x(x)$ is defined as

$$T_x(x) = b_1(L-x)\big\{\omega^2\left[R_0 + \tfrac{1}{2}(L+x)\right] + \xi_0\nu^2\sin\nu t\cos\omega t\big\}$$

and it corresponds to systems stiffening/softening resulting from both transportation motions, while $T_r(x) = \frac{B_4+B_5}{m_0\beta}T_x(x)$. Quantity m_0 is mass of the beam per its unit length and β is a perimeter of the cross-section.
Next, the system is converted to the dimensionless notation and the final ordinary differential equation of motion is written as follows:

$$\ddot{q} + \zeta_1\dot{q} + (\alpha_{11} + \alpha_{13}\Omega_r^2)q + \alpha_{14}\Omega_r\dot{q} - \alpha_{1p}q\sin\Omega_t\tau\cos\Omega_r\tau - \alpha_{1e}\sin\Omega_t\tau\sin\Omega_r\tau = 0, \tag{5}$$

where q is the generalised coordinate corresponding to the studied coupled flexural-torsional motion. Coefficients α_{ij} result from Galerkin's projection, ζ_1 is damping coefficient; Ω_r and Ω_t are dimensionless frequencies of system rotation and to-and-for motions respectively.
Equation (5) is a second-order differential one and it contains the parametric excitation term (with $\sin\Omega_t\tau\cos\Omega_r\tau$) as well as the time dependent excitation one ($\sin\Omega_t\tau\sin\Omega_r\tau$) both resulting from the rotational and the translational motion of the beam. This form of governing equation is different from typical Mathieu-Hill's equations as often met in engineering problems – e.g. column buckling under time periodic compression or pendulums systems.

On-going studies

The given above Eq. (5) inhomogeneous differential equation with periodic parametric and external forcing terms will be solved. The multiple time scales method will be used to determine the boundaries of dynamic stability regions. Discussion regarding selected structural parameters like reinforcing fibers orientation angle and hub radius to beam length ratio will be presented.

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