Sympodial fractal structures: tree-inspired concept for biomimetic engineering design

Ivana Kovacic*, Dragi Radomirovic** Dusan Arsic* and Miodrag Zukovic*
*University of Novi Sad, Faculty of Technical Sciences, CEVAS, Serbia
**University of Novi Sad, Faculty of Agriculture, Serbia

Summary. In this work, the analysis of a tree-inspired fractal structure is presented. First, trajectories of points on branches of different hierarchy of a tree that oscillates freely are recorded in experiments. Then, the principle axes and ellipses of displacements are determined analytically for a sympodial fractal mechanical model. Modes of oscillations and their frequencies are found numerically for different hierarchy and for a constant and cheangable modulus of elasticity along the branches of different order.

Introduction

Trees have been recognized as inspiring concept generators for improved or innovative solutions in engineering given the fact that they: i) blur the boundary between a structure, material and mechanism [1], and ii) cope reasonably well with large amplitude oscillations caused by different types of excitation [2]. Related biomimetic solutions, which copy the concept from nature to man-made structures, have been used in architecture and engineering statics for centuries: first for a decorative purpose, but today also to improve the efficiency of their design realizations given their capacity to carry a large surface supported by a narrow element (trunk) through fractal-like branching configurations [3]. However, biomimicry of their dynamic behavior has not been developed so much, although there have been some hypotheses that the concept can be suitable utilized for adaptive dynamical systems and vibration mitigation devices. Motivated by these hypotheses, this study is concerned with certain tree-like fractal structures and some oscillatory characteristics of their branches of different hierarchy (positions of principle axes, the ellipse of displacement and the influence of changeable modulus of elasticity), which all represent new results.

Investigations: experimental, analytical and numerical

Experiments: qualitative insight

The object of consideration was a young potted tree whose structure is approximately of Leweenberg's type in 2D [4]. The tree was exposed to pull-and-release tests carried out to cause its free vibrations. The resulting motions were recorded by Vicon 3D, which is a leading state-of-the-art infrared motion tracking system that offers high resolution of spatial displacements of the reflective markers arranged along the trunk and the branches of different hierarchy, as shown in Figure 1a (half side view). The arrangement of markers is presented in Figure 1b (front view), and the trajectories of two of them (P and Q) in the vertical *yz* plane are plotted in Figure 1c. The envelopes of the trajectories can be approximated to narrow ellipses. The longer central axes of these ellipses change their position depending on the branching order and the position on the branch. This implies the change of the position of principle axes. To determine these axes, a tree-like fractal structure is considered subsequently.

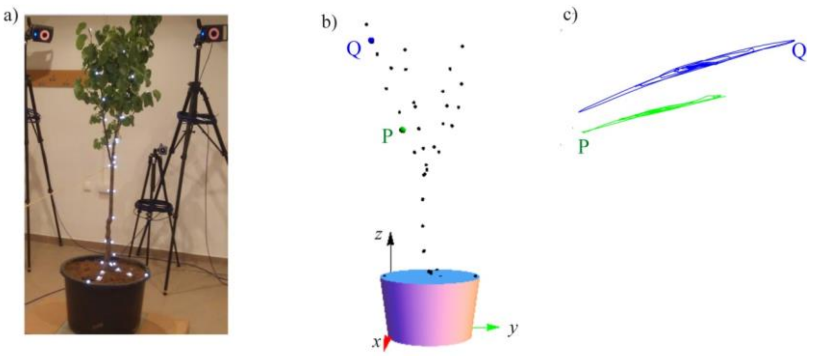

Figure 1. a) Potted tree with markers; b) Markers in graphical presentations; c) Trajectories of the markers P and Q

Analytical approach

The mechanical model is assumed in the form of a sympodial fractal structure, whose geometry and bending stiffness *EI* are shown in Figure 2a,d. The points on the branches oscillate in 2D, and, thus, their discrete mechanical models can be represented as two orthogonal springs, whose position is defined by the position of principle axes. To determine them for all the points along the branches, the concept of two equivalent orthogonal springs is applied [5]. Their position is expressed in terms of the angle φ along which a force **F** acts (Figure 2a,d). The value of this angle leading to a maximal displacement δ_{11} along the branches of the first order is obtained in the form $\varphi_{11} = \frac{1}{2}\arctan\left(\frac{a_1 \sin\alpha + a_2 \sin 2\alpha}{a_0 + a_1 \cos\alpha + a_2 \cos 2\alpha}\right)$ (Figure 2b,c), while the minimal displacement δ_{12} is orthogonal to it

(Figure 2c). For the second-order branches, a maximal displacement δ_{21} corresponds to the angle $\varphi_{21} = \frac{1}{2}\arctan\left(\frac{b_1 \sin\alpha + b_2 \sin 2\alpha + b_3 \sin 3\alpha + b_4 \sin 4\alpha}{b_0 + b_1 \cos\alpha + b_2 \cos 2\alpha + b_3 \cos 3\alpha + b_4 \cos 4\alpha}\right)$ (Figure 2e,f). Note that the coefficients *a* and *b* depend on the position of the point as well as on the geometrical and mechanical characteristics. The mechanical models with two orthogonal springs and the ellipses of displacement with extremal displacements being their half-axes [5] are shown in Figure 2b,c,e,f. They can be related to the trajectories from Figure 1c.

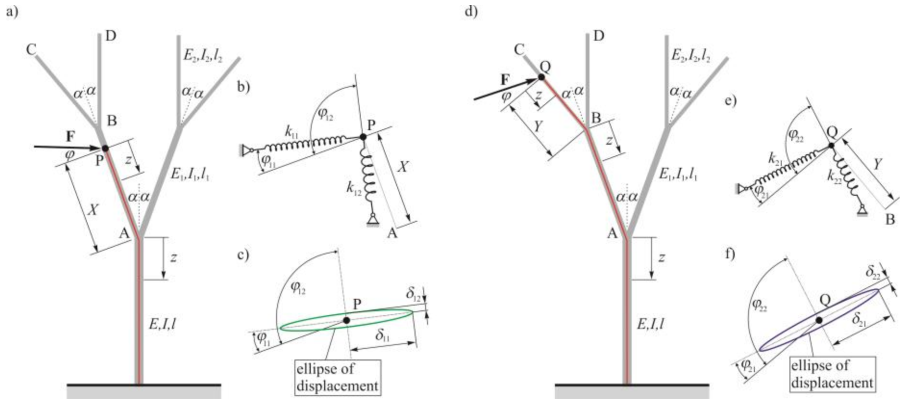

Figure 2. Fractal structure with principle axes for the points on different branches and the ellipses of displacement

Numerical approach

The same structure as shown in Figure 2a is analysed by using a finite element method in AxisVM13 software, but with the branches up to the ninth order. The branching angle is assumed to be α=20^{o}. The slenderness ratio of 3/2 and the lateral branching ratios of 1/2 are used. The modulus of elasticity is first assumed as being constant for the whole structure (*E*=11GPa). Modes of oscillations are presented in Figure 3a for the same fractal structure with a different number of branch order (N=1, 2 and 9). Figure 3b shows how their mode frequencies change for a constant and changeable *E* that decreases for 20% for subsequent branches.

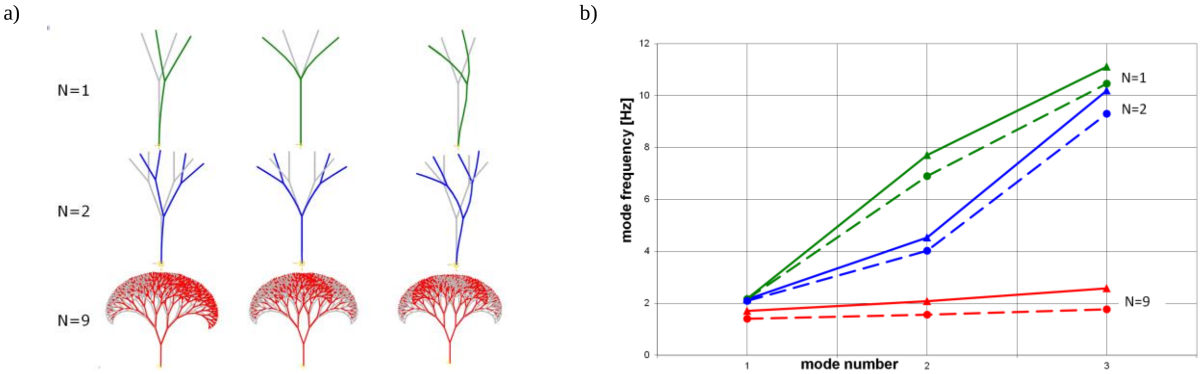

Figure 3. a) Mode of oscillations; b) Mode frequencies for a constant (solid lines) and changeable (dashed lines) *E*

Conclusions

A sympodial fractal tree-like structure has been investigated to determine analytically the positions of principle axes for each point on the branches of the first and second order. Modes of oscillations for different hierarchy have been obtained numerically as well as the corresponding modal frequency for the cases of constant and cheangable modulus of elasticity along the branches of different order. These investigations are still in progress but they have already given some new and interesting insight into the behavior of tree-like structures, which are expected to be benefical for biomimetic applications.

References

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