On the problem of control resonance oscillations of a mechanical system with unbalanced exciters

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Summary. Automatic control system for resonant tuning of a mechanical system with two unbalanced exciters driven by AC motors is discussed in the paper. Proposed control system is based on measurements of phase shift between oscillations of the mechanical system and exciting force unlike other control systems based on amplitudes measurements. Efficiency and high performance of described control system is provided by the usage of so-called “dynamic portrait”, which is a relation between supply frequency of the motors, phase shift and eigenfrequency of the system. Theoretical study is complemented by experimental results. Experimental results proved the efficiency of the algorithm. Advantages and limitations of the proposed algorithm are discussed.

Introduction

Modern vibrating machines (such as vibrating screens) typically operate on above-resonant modes. But it was shown that energy is used in the most efficient way on resonant mode [1]. Thus, resonance tuning can increase performance of vibrating machines and decrease power consumption while improving design and operational qualities of a machine. Practical application of resonant vibrating machines is associated with the problem of resonance mode instability because of nonideal energy source driving vibroexciter and interaction between vibration exciter and oscillatory system. Moreover in real machines operation load is not constant. Changes in operation load could easily break resonant tuning. One possible way to design efficient resonant vibrating machine is to use a control system for continuous resonant tuning.

The most common design scheme of a vibrating technological machine consists of a solid working body set on elastic supports. Oscillations of working body are usually excited by unbalanced vibratos driven by electric motors. There are some researches devoted to control system for machines with DC electric motors. However the issue of stabilizing resonant vibrating machines with unbalanced exciters driven by AC motors (most commonly used) is studied insufficiently so far. Our recent papers were devoted to controls system synthesis for 1DOF system with one exciter [2]. However, real machines have several degrees of freedom and can move in many directions. Moreover, unidirectional exciting forces are usually produced by several synchronously rotating debalances. But interaction of the machine’s oscillatory system with exciters could break synchronization that leads to rotating of exciting force in the plane. In this paper we describe control system for typical vibrating machine with three main degrees of freedom and two unbalanced exciters taking into account interaction of the machine oscillations with exciters.

Design scheme of the machine

In this paper we consider a prototype of vibrating technological machine and corresponding mathematical model which can help us to describe main problems of resonant machine control. Design scheme of a typical vibrating machine with working body having 3 DOF in *y0x* plane and equipped with two debalance exciters is shown in Fig. 1. Working body (also referred as platform further) could be considered as a rigid body on viscoelastic support with linear elastic and damping characteristics. Two unbalanced rotors (exciters) are arranged symmetrically about the axis passing through the center of mass of the machine. Rotors’ axes are parallel to each other and perpendicular to the plane *y0x* (Fig. 1).

Cartesian coordinate system *yOx* is used to describe motion of the machine. Origin of the coordinate system is aligned with static equilibrium position of the platform’s center of mass. Axis *Oy* is directed upwards.

Unbalanced exciters are driven by identical AC motors (most common practical case). The motors are connected to three-phase AC mains via a single frequency converter so that its rotors rotate in opposite directions.

Rotors are driven by torques M_1, M_2. Thus, the system oscillates in the plane *XY*.

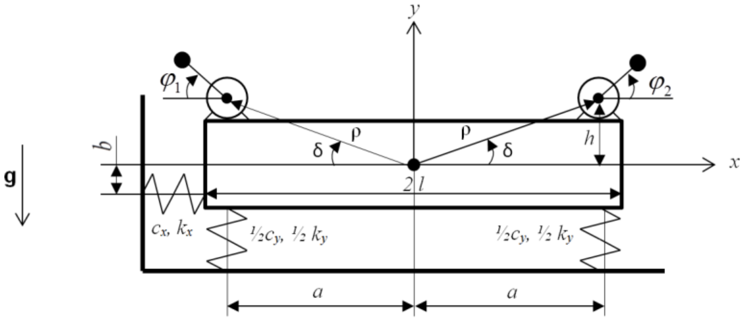

Fig. 1. Design scheme of a vibrating machine

Mathematical model

Motion of the system is described by five generalized coordinates: the linear displacements of the platform's center of mass in the *Ox* and *Oy* directions, rotation angle φ of the platform and rotors' rotation angles φ_1 and φ_2. All angular coordinates mentioned here are measured from the *Ox* axis counterclockwise. Differential equations of motion for the system have been derived using Lagrange equations of the second kind [3]:

$$\begin{aligned}
&m\ddot{x}+k_x\dot{x}+c_x x=m_{r1}r_1\left(\dot{\varphi}_1^2\cos\varphi_1+\ddot{\varphi}_1\sin\varphi_1\right)+m_{r2}r_2\left(\dot{\varphi}_2^2\cos\varphi_2+\ddot{\varphi}_2\sin\varphi_2\right)+\\
&+\left(m_{r1}\rho_1\sin\delta_1+m_{r2}\rho_2\sin\delta_2\right)\ddot{\varphi};\\
&m\ddot{y}+k_y\dot{y}+c_y y=m_{r1}r_1\left(-\dot{\varphi}_1^2\cos\varphi_1+\ddot{\varphi}_1\sin\varphi_1\right)+m_{r2}r_2\left(-\dot{\varphi}_2^2\cos\varphi_2+\ddot{\varphi}_2\sin\varphi_2\right);\\
&J\ddot{\varphi}+k_\varphi\dot{\varphi}+c_\varphi\varphi=\left(m_{r1}\rho_1\sin\delta_1+m_{r2}\rho_2\sin\delta_2\right)\ddot{x}+\\
&+m_{r1}\rho_1 r_1\left[\dot{\varphi}_1^2\sin\left(\varphi_1-\delta_1\right)-\ddot{\varphi}_1\cos\left(\varphi_1-\delta_1\right)\right]+m_{r2}\rho_2 r_2\left[\dot{\varphi}_2^2\sin\left(\varphi_2-\delta_2\right)-\ddot{\varphi}_2\cos\left(\varphi_2-\delta_2\right)\right];\\
&J_1\ddot{\varphi}_1-m_{r1}r_1\left[\ddot{x}\sin\varphi_1-\left(\ddot{y}+g\right)\cos\varphi_1-\ddot{\varphi}\,\rho_1\cos\left(\varphi_1-\delta_1\right)\right]=\sigma_1\left(M_1-M_C\right);\\
&J_2\ddot{\varphi}_2-m_{r2}r_2\left[\ddot{x}\sin\varphi_2-\left(\ddot{y}+g\right)\cos\varphi_2-\ddot{\varphi}\,\rho_2\cos\left(\varphi_2-\delta_2\right)\right]=\sigma_2\left(M_2-M_C\right).
\end{aligned}\tag{1}$$

Here: m_{r1}, m_{r2} – unbalanced masses of rotors, r_1, r_2 – eccentricities of unbalanced masses, J_{r1}, J_{r2} - moments of inertia for unbalanced rotors; $m=m_0+m_{r1}+m_{r2}$ - full mass of the system; m_0 - mass of the platform; k_x, k_y, k_φ - damping coefficients of supports in a horizontal, vertical and angular directions respectively; c_x, c_y, c_φ - stiffness coefficients of the supports in horizontal, vertical, and angular directions respectively; ρ_1, ρ_2 - distance from the platform's center of mass to the axes of rotors respectively; δ_1, $\delta_2=\pi-\delta_1$ - angles between the *x* axis and the axis, which pass through platform's center of mass and axis of rotors in plane *y0x* (counted counterclockwise), and $\delta_1=arctg\left(\frac{h}{a}\right)$, $\delta_2=\pi-arctg\left(\frac{h}{a}\right)$, where *h* is distance between the axis of rotor and axis *Ox*; $2a=2l$ is distance between springs, $b=0$ (see Fig. 1); $J=J_0+m_{r1}\rho_1^2+m_{r2}\rho_2^2$ - moment of inertia of the system; J_0 - moment of inertia of the platform; g - gravitational acceleration; $\sigma_1=+1$, $\sigma_2=-1$ - constants that define the direction of rotors' rotation; M_C - resistance moment for the rotors.

Torques M_1, M_2 in right parts of Eq. (1) could be described by static characteristic of motors. These characteristics are obtained using simplified Kloss formula:

$$\begin{aligned}
&M_1=M_1\left(s_1\right)=\frac{2M_{cr1}}{s_1/s_{cr1}+s_{cr1}/s_1},\\
&M_2=M_2\left(s_2\right)=\frac{2M_{\kappa p2}}{s_2/s_{cr2}+s_{cr2}/s_2},
\end{aligned}\tag{2}$$

where M_{cr1},M_{cr2} - critical (maximum) torques for each motor, s_{cr1},s_{cr2} - slip at critical torque, $s_1=1-P\left|\dot{\varphi}_1/f\right|$, $s_2=1-P\left|\dot{\varphi}_2/f\right|$ - current slip determined by frequency f and angular velocity of rotors $\dot{\varphi}_1$, $\dot{\varphi}_2$, $P=2$ is number of poles pairs.

Thus, equations (1)-(2) describe the system and allow us to simulate dynamics of the system taking into account interaction of the machine oscillations with exciters. Certain solutions of these equations will be used in control system.

Control system synthesis

Although the system has 3DOF and 3 resonant modes respectively, we consider control system to tune machine to near-resonant mode of vertical oscillations only as the most practically important case. The main problem is that in real machines there are fluctuations of the full mass of the system due to changes in operation loads. But it is difficult to measure these changes directly. So the purpose is to develop algorithm that allows to control the system without direct measurement of full mass of the system or operation load.

Usually it is proposed to establish resonance mode by measuring oscillation amplitude. However, in this case we need to know all parameters of the system in advance that contradicts to considered problem statement. Here we propose control algorithm based on phase shift measurement.

More specifically, developed control algorithm is based on the fact that phase shift $\Delta\varepsilon$ between platform oscillation law $y(t)$ and driving force acting on the platform $F(t)$ depends on current mode. This phase shift $\Delta\varepsilon$ equals to $\pi/2$ in resonant mode [4]. So the key idea of the algorithm is to tune controlled parameter to reach phase shift $\Delta\varepsilon = \pi/2$. The most convenient way to control AC motors is to use frequency converter. So it is naturally to use power supply frequency as controlled parameter. To compute current phase shift we have to collect data that describes system dynamics. It is proposed to compute phase shift from experimental data: platform displacement and rotors' angular positions. Platform displacement could be tracked by accelerometers attached to the platform. Rotors' angular positions could be measured by rotatory encoders. For any moment t_i, when the platform is in the static equilibrium position, phase shift is determined by the formula $\Delta\varepsilon_i = \varphi^* - 2\pi n$, here $\varphi^* = (\varphi_1 + \varphi_2)/2$ is angle of total driving force direction, n is number of full revolutions in φ^*. It should be mentioned that this measurement method is applicable for steady state modes only. Steady state should be established by phase shift variance analysis: system is considered to be in steady mode if phase shift variance does not exceed some predefined threshold value (1° for instance).

So the basic idea of the control algorithm is to increase power supply frequency if measured current phase shift is below $\pi/2$ and decrease it otherwise. To determine necessary change in supply frequency in this paper we propose to use pre-computed characteristic of the system which we call "dynamic portrait".

Essentially dynamic portrait is the relationship of phase shift Δε with the control parameter (supply frequency $f_n = \omega_n/(2\pi)$) and the natural frequency of vertical oscillations Ω. It can be represented graphically as a three-dimensional surface (see Fig. 2). Shape of the surface depends on parameters of the machine. Dynamic portrait for a particular machine supposed to be computed from solutions of equations (1)-(2) by slow increase in supply frequency on a set of different values of system mass.

Dynamic portrait gives the new value of supply frequency that corresponds to resonant mode for any current state of the machine. Thus control algorithm will use dynamic portrait in the following way. First of all it is necessary to define current state of the machine. Current power supply frequency is known at any moment because it is controlled parameter. Current phase shift could be computed as it was described above. These two parameters determine a point on the surface (point B in Fig. 2) that represents current state of the system. On the other hand this point gives us unknown natural frequency (or mass) of the system. This estimation of natural frequency allows us to determine resonant state of the system where phase shift $\Delta\varepsilon = \pi/2$. This resonant state is represented by point C on line A (see Fig. 2) that represents all possible resonant states ($\Delta\varepsilon = \pi/2$). So new resonant frequency could be determined as corresponding coordinate of point C.

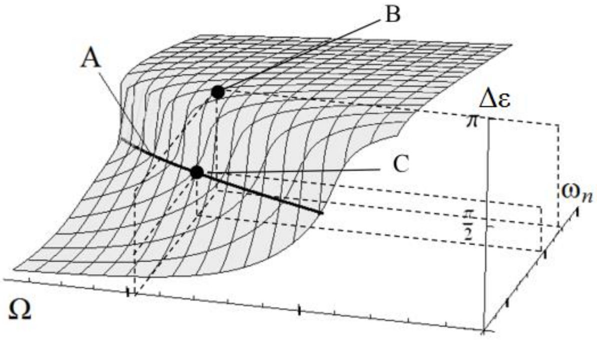

Fig. 2. Graphical representation of dynamic portrait

Designed control system is based on the feedback principle. The diagram of the control system is shown in Fig. 3.

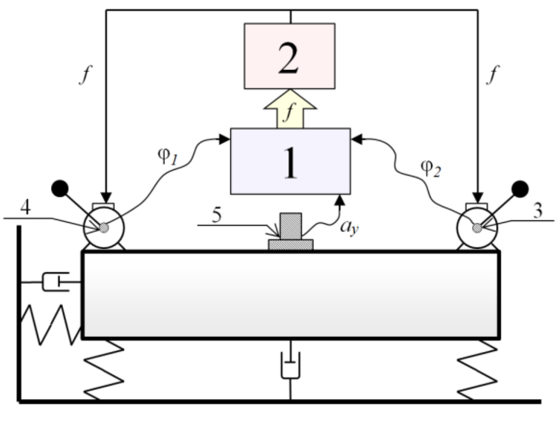

Fig.3. Scheme of the control system

Supply frequency is controlled via frequency converter *2*, which is common for two AC motors. The frequency converter is controlled by control unit *1*. The control algorithm implemented on control unit computes value of the controlled parameter which is supply frequency *f* using dynamic portrait and value of phase shift measured using rotatory encoders *3*, *4* and accelerometers *5*.

Experimental research

To prove claimed theoretical statements we designed an experimental setup that corresponds to described scheme of machine. The experimental setup has been built in laboratory of Vibrational Mechanics in Mechanical Engineering Research Institute of the Russian Academy of Sciences. The setup is shown in Fig.4. It consists of a solid platform set on 14 springs. Two AC motors with unbalanced rotors are attached to the platform. The setup is equipped with accelerometers and rotatory encoders (aren't visible in the figure) as in the scheme described above. Control algorithm has been implemented on FPGA device (NI cRIO-9064). Frequency converter (Mitsubishi FR-D720) has been used to control motors by control unit. This prototype supposed to be symmetrical to correspond to mathematical model described above.

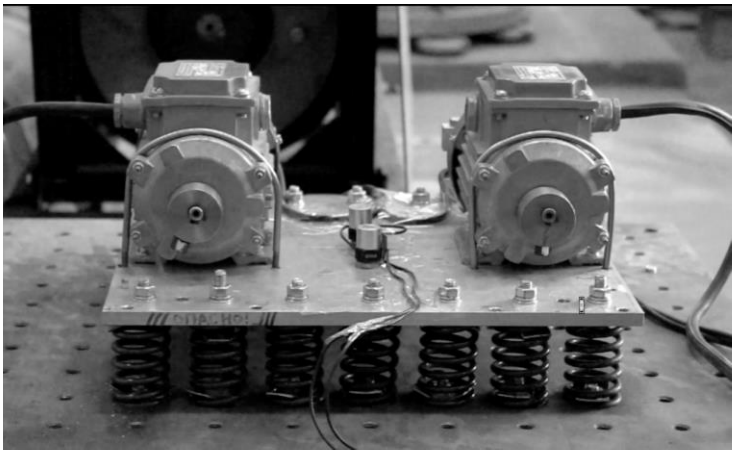

Fig.4. Experimental setup

All necessary parameters of the experimental setup have been measured (parameters like spring stiffness, dimensions, etc.) or estimated indirectly (damping coefficients, etc.). Dynamic portrait of the system has been computed substituting all this parameters to equations (1)-(2). Note that taking into account 3DOF and self-synchronization of the exciters allows to find ranges of supply frequencies with necessary synchronization of debalances, which form the operating range of the control system. In the remaining frequency regions, stable synchronization is either absent or another type of synchronization is realized which does not provide the required oscillation form. When the machine is operating in these frequency ranges, the control algorithm described above does not allow determining the phase shift, so these frequency ranges were excluded from consideration in the experimental study.
The idea of the experiment is to find out if designed control system could tune the prototype to resonant mode. Results of the experiment are shown in Fig. 5 as time diagrams of phase shift Δε and power supply frequency *f*.

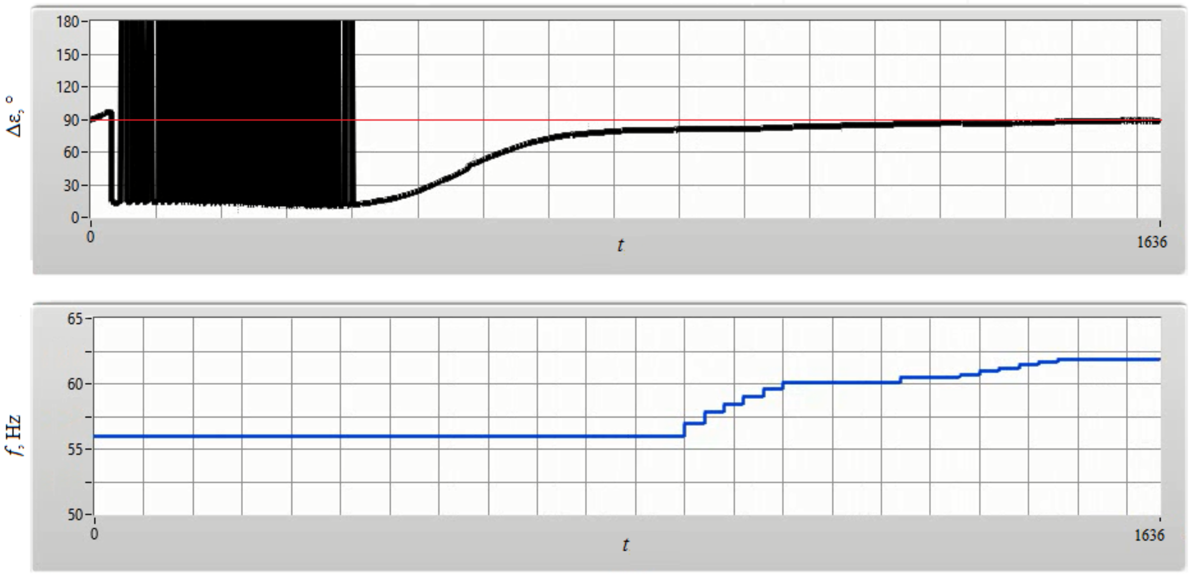

Fig.5. Experimental results

Initially the prototype starts to accelerate from stationary state. Supply frequency is set to a relatively small value so the machine appears to be in below-resonant mode. Then automatic control system switches on. Control system operation is based on phase shift measurement, but phase shift could be measured correctly only in stable mode as it was mentioned above. Obviously oscillation mode is not stable during acceleration. So control system waits for stable oscillation mode. Incorrect phase shift measurements are shown in Fig. 5 as solid black area. After the transition process is finished control system starts to tune power supply frequency. Theoretically only one step of frequency change is needed. In the experiment there were 13 steps as it could be seen in the figure. It can be explained by several reasons. The most important reasons are errors in experimental measurements of phase shift and system parameters. Also it’s necessary to note that errors in estimation of damping coefficients significantly impact on dynamic portrait. Nevertheless control system tuned the prototype to resonant mode successfully in less than 15 seconds. In Fig. 5 $\Delta\varepsilon=90°$ corresponds to resonant mode.

Conclusions

Experimental results obtained on controlled vibrating machine prototype proved that the developed control system is able to tune a vibrating machine to vertical oscillation resonance. Operating range of the control system is limited by frequencies at which both stable oscillations and necessary synchronization of exciters are realized, and is mainly determined by conditions of self-synchronization of the exciters. All parameters must be measured accurately to take full advantage of the dynamic portrait.

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