Analysis of stability transitions in a microswimmer with superparamagnetic head

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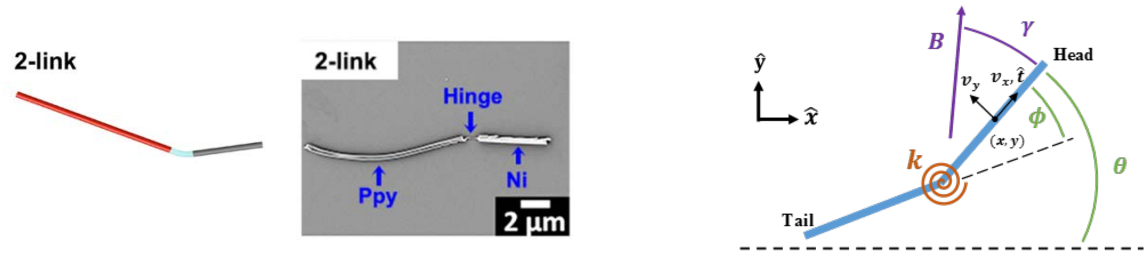
Summary: Robotic microswimmers are a source of growing interest in the fields of physics and biomedical robotics. The famous work of Dreyfus *et al* (2005) [1] introduced a robotic microswimmer composed of a chain of superparamagnetic beads and actuated by a planar oscillating magnetic field. Further experiments and numerical simulations of the swimmer model [2,3] revealed that for large enough oscillation amplitude of the magnetic field's direction, the swimmer's mean orientation and net swimming direction both flip from the mean direction of the magnetic field to a direction perpendicular to it. In the current work, this phenomenon is analyzed theoretically by studying the simplest possible microswimmer model: two slender rigid links connected by an elastic joint, while one link is superparamagnetic. The dynamic equations of motion are formulated explicitly, and various asymptotic analyses are carried out, leading to analytical conditions for stability transitions and explicit expressions of the swimmer's mean speed, both confirmed via numerical analysis of the model. It is found that there exist intermediate parameter regions of dynamic bi-stability where the solutions of swimming about the aligned and perpendicular directions are both stable under different initial conditions. Since the system can be simplified as a 2nd order ODE with parametric excitation, the stability transitions are reminiscent of those exhibited by Kapitza's pendulum with an oscillating pivot.

Background

Microswimmers are creatures, biological or robotic, that can move in a fluid by changing their body's shape. Their motion is characterized by the absence of inertia, as their Reynolds number is extremely low. Various micro-organisms such as bacteria and sperm cells have inspired the manufacturing of robotic microswimmers. The motivation for investigating robotic micro-swimmers is their potential use in various biomedical applications such as biopsies, targeted drug delivery and even minimally invasive surgeries. One of the most promising methods of actuating microswimmers is by applying a time-varying external magnetic field. A simplified theoretical model of such a swimmer, made of two ferromagnetic links and actuated by an oscillating magnetic field, was studied thoroughly in [4] and the results were verified in experiments [5]. The model of the swimmer and a fabricated swimmer are shown in Figure 1.



(a) (b) (c)

Figure 1 – (a) A schematic model of the swimmer. (b) A fabricated ferromagnetic swimmer [5]. (c) Our model of the microswimmer, based on the model proposed in [4]. The difference between the models is in the magnetic properties of the "head" link.

However, in the famous work published by Dreyfus et al. [1], the magnetic properties of the swimmer were not ferromagnetic, but rather superparamagnetic. That is, the material has zero remnant magnetization but it polarizes in response to application of an external magnetic field. A theoretical model of a superparamagnetic micro-helix generating corkscrew-like propulsion has recently been analyzed in [6]. However, a simple analytical model of a microswimmer with superparamagnetic head under planar oscillations as in [1] has not yet been considered.

Problem statement and asymptotic limits considered

Proposed theoretical model

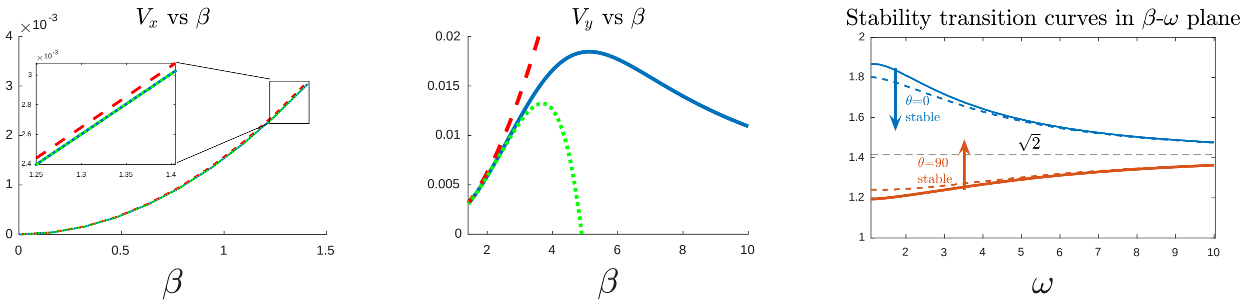
Our model is based on the model proposed in [4], with only the magnetic properties changed. The microswimmer consists of two slender links, connected by a torsion spring that generates a torque proportional to the relative angle ϕ between the links. The head link of the microswimmer is made of superparamagnetic material having uniaxial anisotropy with an easy axis along the link's longitudinal axis. A uniform, oscillating external magnetic field of the form $\mathbf{B}(t) = b\left(1, \quad \beta\sin(\omega t)\right)^T$ is applied and generates a torque on the magnetic link. The microswimmer is submerged in a viscous fluid and since its Reynolds number is extremely small, the viscous drag forces can be calculated using resistive force theory. The swimmer's inertia is neglected, hence it is always in force and torque balance. The resulting equations of motion are a set of nonlinear, time-periodic, 1st order ODEs. The microswimmer's model is shown in Figure 1 (c).

Fast actuation frequency

The first asymptotic limit we consider is when the actuation frequency is extremely high. By using the method of multiple scales we are able to separate the slow dynamics of the swimmer, that show the swimmer's mean orientation, from the fast dynamics, that show undulations and the net motion generated by the swimmer. The equations of the slow dynamics show that the stability transition between swimming along x axis to swimming along y axis is dependent solely on the ratio between the amplitudes of the x and y terms of the magnetic field, β and occurs when $\beta = \sqrt{2}$, in agreement with [2]. From the equations of the fast dynamics we can find approximations of the swimmer's mean speed, and it is found that there exists an optimal ratio β that maximizes the swimming speed, which is often attained when swimming along the y direction (see Figure 2 (a) and Figure 2 (b)).

Fast actuation frequency and a stiff swimmer

The second asymptotic limit we consider is the case of a stiff swimmer and high actuation frequency. In this case, the equations of motion can be reduced into a single nonlinear 2nd order ODE that resembles the equations of a pendulum with a pivot that oscillates at an angle relative to gravity (inclined Kapitza's pendulum). This swimmer's equation has two periodic solutions, oscillating about either x direction or y direction. Considering small perturbations about a periodic solution, a linear 2nd-order time-periodic variational equation is formulated, that has the form of Hill's equation. We approximate the oscillating solutions by linearizing the 2nd order ODE and using harmonic balancing, and the obtained approximation is then substituted into the variational equation. Using the method of Hill's infinite determinant, stability limits of each solution are obtained analytically. It is found that the stability limits are dependent on all of the physical parameters of the swimmer in a rather complicated manner. Moreover, *bistability* regions where both solutions are stable are observed (see Figure 2 (c)).



(a) (b) (c)

Figure 2 – (a) and (b): Mean speed in x and y directions, respectively. The solid line represents numerical simulations, the dashed line is the analytical approximation up to 3rd order and the dotted line is the analytical approximation up to 5th order. (c) Stability limits of each solution in terms of the magnetic field's properties, for a specific set of swimmer's physical properties. The solid lines are the limits obtained analytically and the dashed lines are the limits obtained from numerical simulations.

Conclusions

We have shown that a microswimmer with superparamagnetic properties exhibits transitions between two solutions, the stability limits of each solution were obtained and regions of bi-stability were found. We also observed an optimum of the swimmer's speed in β, and have shown that sometimes swimming along y axis is faster than swimming along x axis. All of the results were compared with numerical simulations and have shown good agreement.

References

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