Control of nonlinear localized waves by an external action

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Summary. A link between the action on the lateral surface of a wave guide and the parameters of the control function in nonlinear wave equations describing strain wave propagation inside the wave guide, is studied asymptotically. The control provides stable localization of nonlinear strain waves with desired shape and velocity. The link allows us to justify recently found control functions and to find out the kind of the external action required for the support of desired localized strain wave.

Statement of the problem

In a series of papers [1, 2, 3] we have demonstrated an efficiency of a control for support of localized wave solutions to some nonlinear equations. The idea of the control is in an addition of a term which governs a tendency to the wave with desired shape and velocity. In particular, the double sine Gordon was modified in [3],

$$U_{tt} - U_{xx} + \sin(U) + q\sin(2U) + u(x,t) = 0. \tag{1}$$

where control function $u(x,t)$ provides a tendency to a desired wave solution $U^*(x,t)$ that may be chosen in the form of both the bell-shaped and the kink-shaped wave propagating with some prescribed velocity. However, a natural question arises how to find out a physical meaning of the additional term in the equation.

The equations studied are the 1D nonlinear PDEs. Sometimes they appear as a result of an asymptotic reduction of an initially 3D problem. In particular, the equations account for strain waves propagation in a wave guide usually with stress-free lateral surfaces. In this paper an attempt is made to consider various kinds of the lateral surface stress actions in order to find out those that result in arising of additional terms in the reduced equations in the form similar to the from of the control function. It allows us to establish a connection between the parameters of the surface stresses and the parameters of the control function. Derivation of highly nonlinear nonlinear equations like the sine- Gordon equation and its generalization will be studied. An asymptotic technique allows us to reduce the original 3 D equation to the one model 1D equation in the long wave limit.

References

[1] Porubov, A.V., Fradkov, A.L., Bondarenkov, R.S. and Andrievsky, B.R. (2016) *Commun. Nonlinear Sci Numer Simulat* **39**:29-37.

[2] Porubov, A.V., Fradkov, A.L. , Andrievsky, B.R., and Bondarenkov, R.S. (2016) *Wave Motion* **65**:147-155.

[3] Porubov, A.V., Antonov, I.D., Fradkov, A.L. , Andrievsky, B.R. (2016) *Intern. J. of Non-Linear Mech.* **86**:174Ű184.