A nonlinear model for design of beams operating in largely deformed configurations

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Summary. Strings and beams are ubiquitous in engineering applications and even in nature. A model with proper constitutive laws is deduced and applied for studying nonlinear dynamics of beams undergoing very large displacements accompanied by small local strains, with possible target applications found in modern engineering fields. Results are compared with available existing literature.

Introduction

Strings and cables or rods and beams are ubiquitous among man-made products and, consequently, models for their analysis are currently met in any field of civil and industrial engineering. Further, the application of beam models is not restricted to bodies having a prevalent dimension with respect to the others, but is extended to structures that can suitably resemble beams, at least at a proper "distance" of observation or at a certain level of investigation. An example is represented by so-called beam-like lattice structures [1].

The analysis of behavior of beams with low, high or even very high slenderness, which is the ratio between the length of the beam and the smallest gyration radius, is an important field of application of any beam theory. For instance, slender beams are used as members of buildings and, depending on specific technical codes, they must be designed to face nonlinear issues as buckling, plastic deformations and local failures. In aeronautics, aircraft wings and helicopter rotor blades, which are often made of composite materials, are subjected to nonlinear loads and may undergo moderately large deflections. In manufacturing industry, fishing rods are designed to undergo very large displacements with small recoverable deformations. Finally, needless to say, beams and beam-like structures are not confined to man-made world, but they are easily seen in nature. For instance, at a very basic level of the analysis, thus neglecting complexities in description of their internal structure, tendon spindles can be roughly treated as cables and long bones, as femurs, can be studied as beams.

The proposed contribution is aimed at the study of dynamics of Cosserat-like beams, equipped with non-standard constitutive laws. The beams are assumed undergoing large deflections with small local strain. Although, our primary interest and motivation are in the analysis of the model itself, we emphasize that a deeper knowledge of behavior of highly flexible one-dimensional bodies can guide their design to better fulfill specific needs or reach assigned targets. Indeed, the model under investigation can drain large interest in modern industrial fields. To cite just but a few, geometrically exact beam models have been recently applied in the emerging field of soft robotics, to describe behavior of continuum arms driven by cables [2] and in computer graphics to simulate threads, ropes, cables or hair strands [3].

The model

We consider initially straight beams undergoing planar and twist-less deformed states. We assume that cross sections remain flat and undistorted, but not necessarily orthogonal to the beam axis during the deformation process. Figure 1, to which we refer for notations, shows a slice, which has lengths $\mathrm{d}x$ in the undeformed configuration and $\mathrm{d}s$ in the deformed one. In the following, $u(x,t)$ and $v(x,t)$ stand for the axial and transversal displacements of the beam axis, $\varphi(x,t)$ and $\theta(x,t)$ are the beam axis rotation and the cross-sectional rotation, respectively. We assume that displacements (and forces) are positive if concordant with the reference (see the two vectors in Figure 1) and rotations (and moments) are positive if counterclockwise oriented.

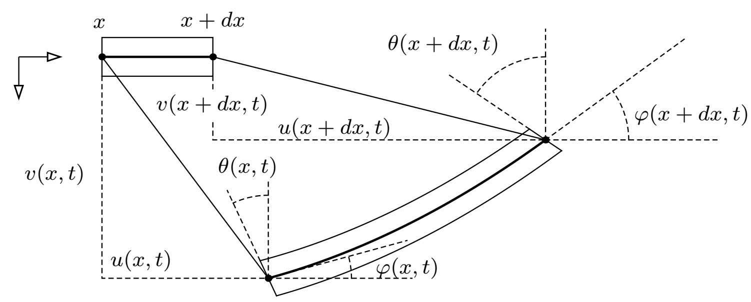

Figure 1: Element in reference and deformed configurations.

Equations of motion

On introducing the axial strain ε, the shear angle γ and the lagrangian bending curvature κ, non-standard constitutive laws for axial and transversal forces N and T and for the bending moment M can be written (see [4] for details on their derivation) as

$$N = \frac{\sin^2\gamma}{\sqrt{2\varepsilon+1}}\,G\,A + \frac{\varepsilon}{\sqrt{2\varepsilon+1}}\,E\,A + \frac{1+2\cos^2\gamma}{2\sqrt{2\varepsilon+1}}\kappa^2\,E\,I + \frac{\cos\gamma}{2(2\varepsilon+1)}\kappa^3\,E\,\mathcal{I}\,, \tag{1}$$

$$T = \sqrt{2\varepsilon+1}\sin\gamma\,\cos\gamma\,G\,A - \sqrt{2\varepsilon+1}\sin\gamma\,\cos\gamma\,\kappa^2\,E\,I - \frac{\sin\gamma}{2}\kappa^3\,E\,\mathcal{I}\,, \tag{2}$$

$$M = \left(\varepsilon + (2\varepsilon+1)\cos^2\gamma\right)\,\kappa\,E\,I + \frac{3}{2}\,\sqrt{2\varepsilon+1}\,\cos\gamma\,\kappa^2\,E\,\mathcal{I} + \frac{1}{2}\,\kappa^3\,E\,\mathbb{I}\,, \tag{3}$$

and the geometrically exact equations of motion are stated as

$$\rho\,A\,\ddot{u} + c\,\dot{u} \;=\; \left(N\frac{\mathrm{d}s}{\mathrm{d}x}(1+u') - Tv'\left(\frac{\mathrm{d}s}{\mathrm{d}x}\right)^{-1}\right)'\,, \tag{4}$$

$$\rho\,A\,\ddot{v} + c\,\dot{v} \;=\; \left(N\frac{\mathrm{d}s}{\mathrm{d}x}v' + T(1+u')\left(\frac{\mathrm{d}s}{\mathrm{d}x}\right)^{-1}\right)'\,, \tag{5}$$

$$M' + \rho\,I\,\ddot{\theta} \;=\; T\sqrt{2\varepsilon+1}\,, \tag{6}$$

where ρ is the mass density, c is the damping coefficient, E is the Young's modulus, G is the (second) Lamé's constant, A, I, $\mathcal{I}$ and $\mathbb{I}$ are the cross-sectional area, the second, the third and the fourth moment of area, respectively, and a superimposed dot denotes derivative with respect to time t and a prime stands for derivative with respect to x.

A sketch on numerical results

Equations (4-6) are integrated numerically applying, as discretization schemes, finite differences in space and variable step-size fourth-order Runge-Kutta in time. Figure 2 reports results in the static case [4] for two cantilever beams, one inextensible (a), the other extensible (b). Results perfectly agreed with classical findings [5].

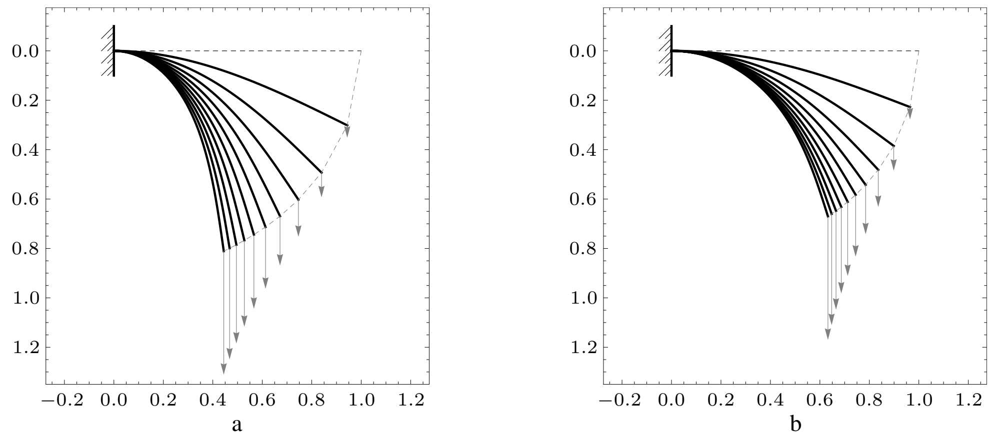

Figure 2: Deformed shapes of the inextensible (a) and extensible (b) cantilever beams. Displacements are assumed positive if oriented from left to right (horizontal) and downward (vertical). Notice that displacements are not amplified.

Conclusions

A Cosserat-like beam model is deduced. The beam is supposed undergoing very large displacements, while local strain remains small enough and completely recoverable to consider the material in its purely elastic phase. The highly nonlinear behavior of the beam is investigated, both in statics and dynamics. Results are compared with available findings reported in the scientific literature.

References

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